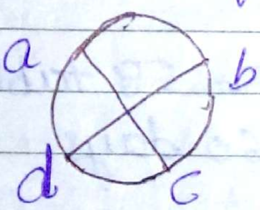
— Graph Theory —

Graph theory →
- Graph represented by G(V, E) is a collection of finite vertices and finite edges.

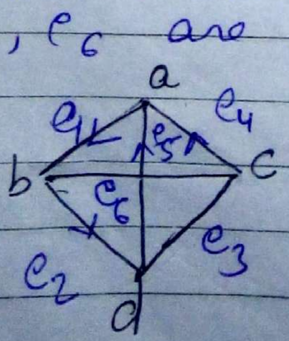
Directed Graph —
- Directed Graph a graph whose direction is fixed or we can say that a graph which has a starting point and end point.

Undirected graph →
- If the direction of an edges is not define a graph that graph is called undirected graph.



Mixed Graph →
- A graph in which some edges are directed and some edges are undirected is called Mixed graph.

e_1, e_2, e_4, e_5 are directed
e_3, e_6 are undirected



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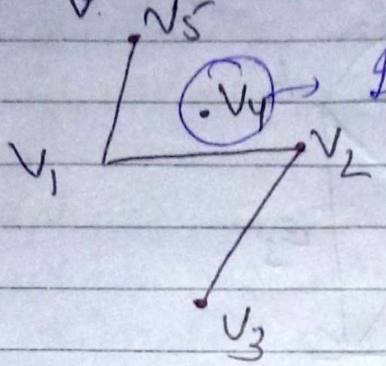
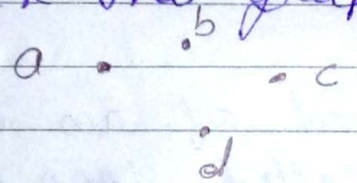
(4) Isolated Graph →

- A graph in which a vertex which is not Connected with any edge is called Isolated Vertex or vertex of 0 degree.



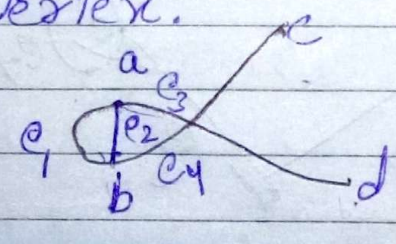
(5) Null Graph →

- A graph in which each vertex is an isolated vertex that graph is called Null Graph.



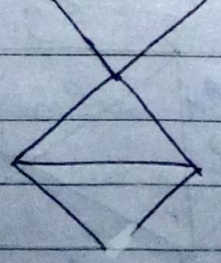
(6) Pendent Vertex —

The graph in which vertexes is associated with single edges that vertices is called Pendent vertex.



c and d are pendent vertice

Others



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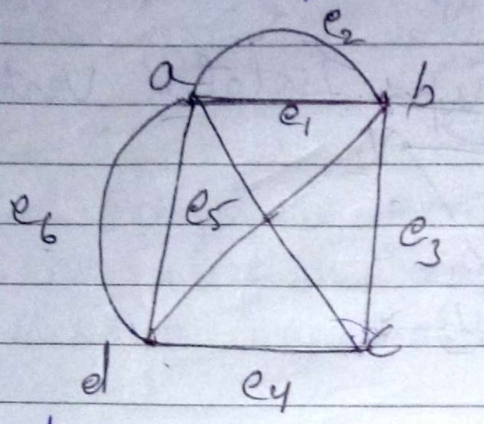
(7) Multiple edges or parallel edges —

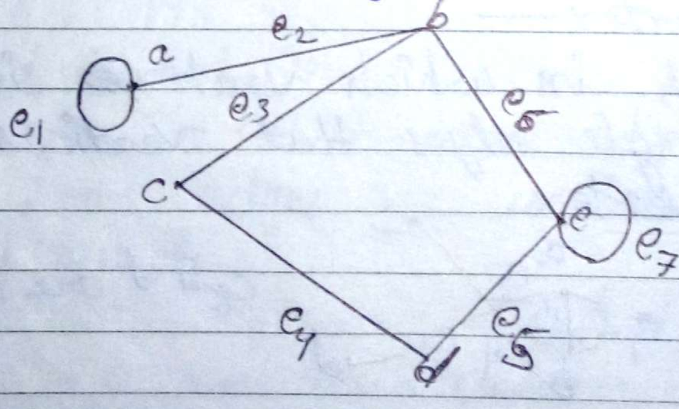
- A graph if their exist more than one edges ~~within~~ betw. any pair of vertices that edges are called multiple edges or parallel edges.



e_1 and e_2 are parallel.
e_5 and e_6 are parallel

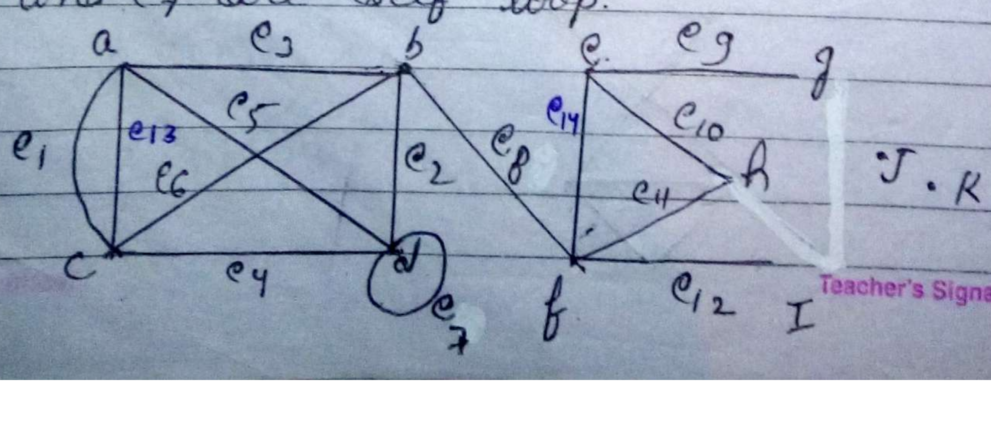
(8) Self loop →

If the origin and end of an edge are same vertex then it is called loop or self loop. It is represented by close curve.



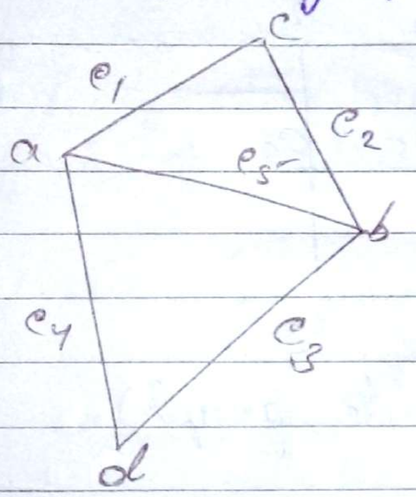
e_1 and e_7 are self loop.

Ques →

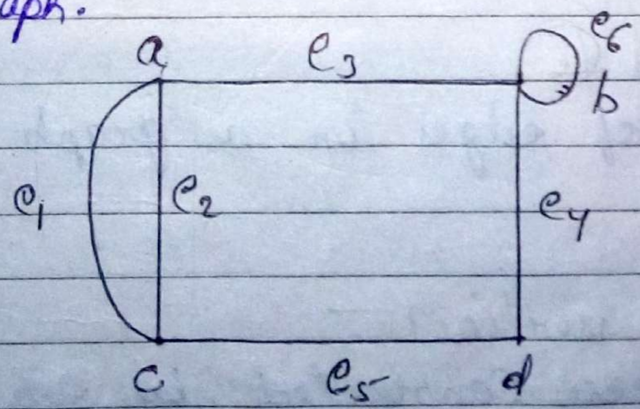


Isolated vertex → J.K
Pendent vertex → g.i
loop → e_7
Parallel edges → e_1, e_{13}
Number of vertices → 11
Number of edges → e_{14}

* Simple Graph →
- A graph which has no loop and parallel edges is called simple graph.



* Multi graph →
A graph which is not a simple graph is called Multi graph.



* finite graph →

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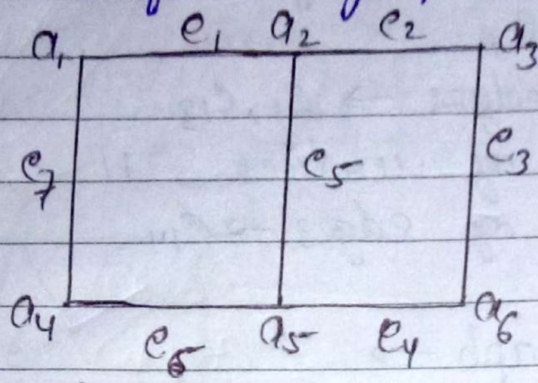
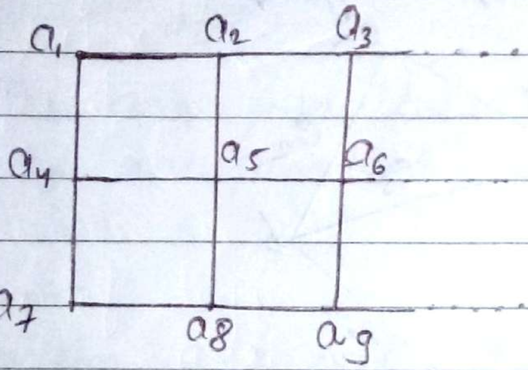
A graph in which no. of vertices and edges are finite is called finite graph, otherwise infinite graph.



finite graph



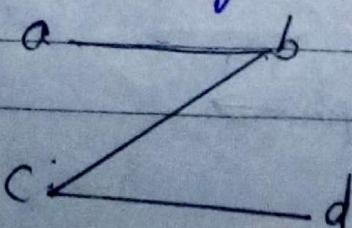
(Infinite graph)

* Order graph →
- The no. of vertices in a graph is called order of graph.

* Size graph →
- Number of edges in a graph is called size of graph.

* Adjacent vertices →

Two vertices connected by an edge is called adjacent vertices.



Adjacent vertices

→ (a,b), (b,c), (c,d)

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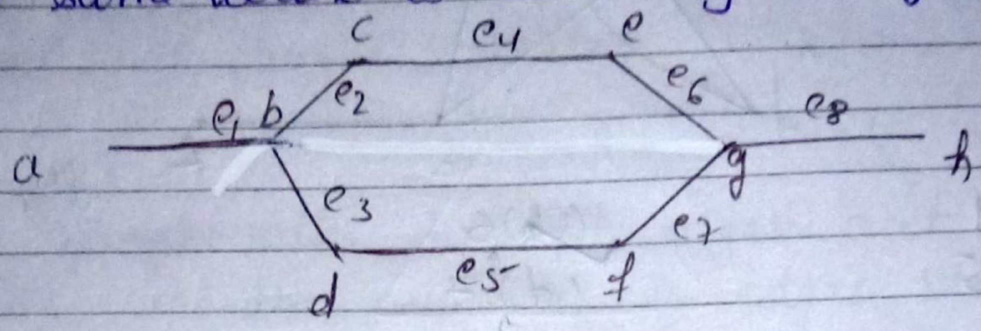
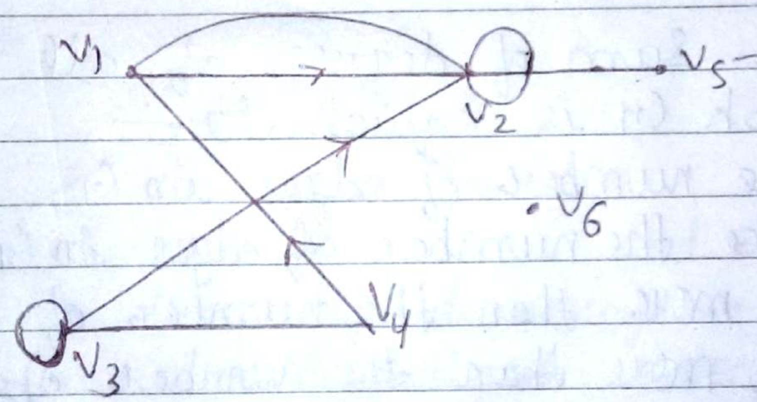
Adjacent Edges →

Two non-parallel edges which are connected to a same vertex is called adject edges.



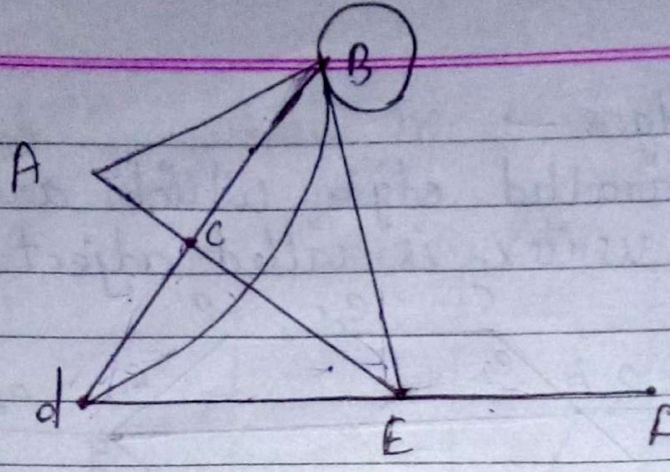
Degree of vertex →

- The number of edges associated with a vertex is called degree of vertex.



Degree of V_1 → 3
Degree of V_2 → 6
Degree of V_3 → 4
Degree of V_4 → 2
Degree of V_5 → 1
Degree of V_6 → 0

Ques In the following graph the degree of vertex of B is ____



graph – G

(a) 4
(b) 5
(c) 6
(d) 7

Ques→ A vertex of degree 0 is called –
(a) Pendent Vertex
(b) point to point vertex
(c) Isolated vertex
(d) Vertex

Ques→ The sum of degrees of all vertices in a graph G is equal to :–
(a) The number of edges in G
(b) Twice the number of edges in G
(c) One more than the number of edges in G.
(d) Two more than the number of edges in G.

Sol^n → No. of vertex in A 2
degree
~~No.~~ of vertex in B 6
No. of vertex in C 4
No. of vertex in D 3
No. of vertex in E 4
No. of vertex in F 1
Sum of these vertices is 20
No. of edges in Graph G is 10.
So, we can say that twice the No. of edges in graph G.

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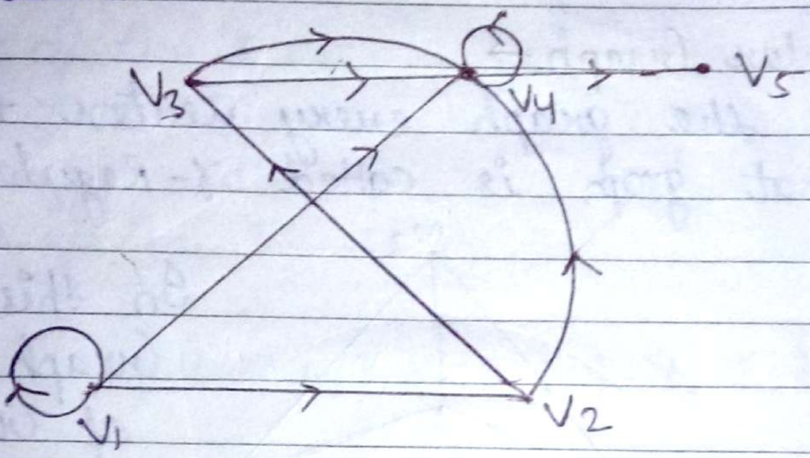
* In degree Graph →

- In a degree with directed edges the in degree of a vertex is the number of edges directed towards the vertex.

* Out degree —

- In a graph with directed edges out degree of a vertex is the number of edges start from the vertex.

Ques →



In Degree	Out degree
In degree of V_1 → 1	Out degree of V_1 → 1
In degree of V_2 → 1	Out degree of V_2 → 2
In degree of V_3 → 1	Out degree of V_3 → 2
In degree of V_4 → 5	Out degree of V_4 → 2
In degree of V_5 → 1	Out degree of V_5 → 0

Ques → In a graph their are seven vertices of each of degree for then no. edges in the graph?

(A) 4 (B) 7 (C) 28 (d) 14

* edges are double of

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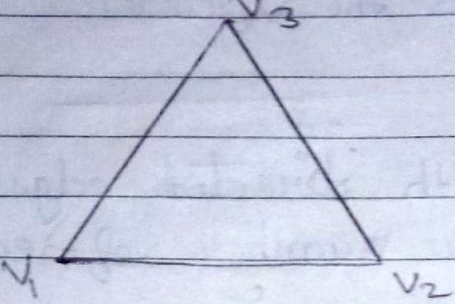
Regular Graph →

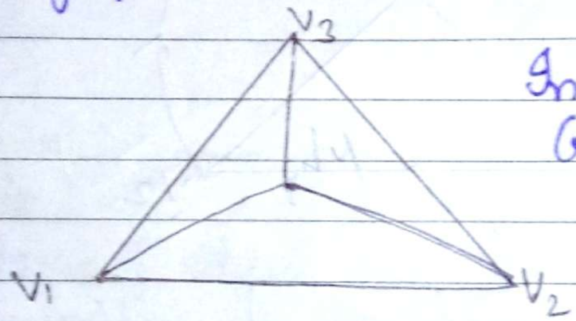
- If degree of each vertex is equal that graph is called Regular graph.



Degree of $V_1 = 2$
Degree of $V_2 = 2$
Degree of $V_3 = 2$

γ - Regular Graph →

- If in the graph every vertex has a degree "γ" that graph is called γ-Regular Graph.



In this Regular Graph Degree of Graph.

Even vertex →

- In a graph if a degree of vertex is even number that is called even vertex.

Odd vertex →

- In a graph if a degree of vertex is odd number that is called odd vertex.

Degree Sequence of Graph →

- The ascending order of degree of vertex is of graph is called Degree Sequence of Graph.

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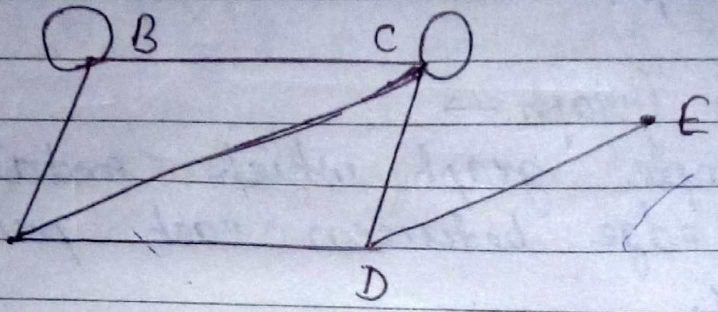
<=> note →

- for at first all we find degree of each vertex and arrange them in ascending order

Ques →



Degree of A = 3
Degree of B = 4
Degree of C = 5
Degree of D = 3
Degree of E = 1
Degree Sequence :- {1, 3, 3, 4, 5}

Ques → find the size of an Regular Graph G(V, E)
Ans → G(V, E) → No. of vertices V and No. of edges E

$$\sum_{j=1} v_j = n + n + n \dots + V \text{ times}$$

$$nV = 2E$$

$$E = \frac{nV}{2}$$

Ques → find the size of three Regular Graph with 10 vertices.
Ans →
Sum of degree of vertices
$\Rightarrow 3 \times 10$
$\Rightarrow 30$

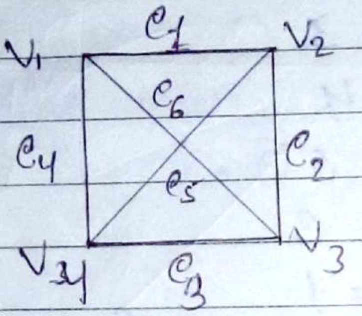
No. of edges = $\frac{\text{Sum of degree of vertices}}{2} = \frac{30}{2} = 15$ Ans

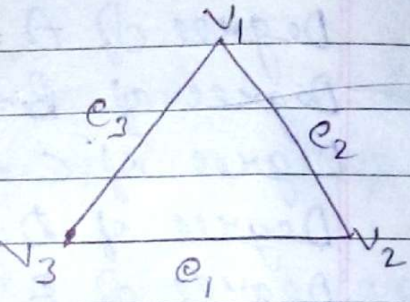
Hand Shaking theorem →

- The sum of degree of all vertices is equal to twice the number of edges in the graph.

Complete Graph →

- A simple graph which contain exactly one edge between each pair of distinct vertex.





$e_1 \rightarrow v_1$ and v_2 | $e_1 \rightarrow v_2$ to v_3
$e_2 \rightarrow v_2$ to v_3 | $e_2 \rightarrow v_2$ to v_1
$e_3 \rightarrow v_3$ to v_4 | $e_3 \rightarrow v_1$ to v_3
$e_4 \rightarrow v_4$ to v_1
$e_5 \rightarrow v_2$ to v_4
$e_6 \rightarrow v_1$ to v_3

Size of K_n

- If graph has n vertices then we connect n vertices with $(n-1)$ vertices. Then no. of edges is equal to $^nC_2 = \frac{n(n-1)}{2}$

$n \rightarrow$ No. of vertex

Ques→ Find the size of graph having 10 vertices each of degree 5.

Ans→ $\frac{n(n-1)}{2} \Rightarrow \frac{10(10-1)}{2} \Rightarrow \frac{\cancel{10}^{5} \times 9}{\cancel{2}} \Rightarrow 45$

Ques- find that five regular graph with 13 vertices is possible.

An→ $E = \frac{nv}{2}$ — why we will use this formula in this condition?

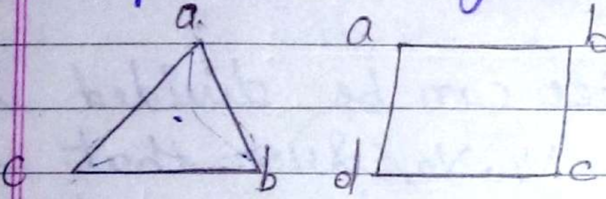
$= \frac{5 \times 13}{2} = \frac{65}{2} = 32.5$

Ques- find the regularity of complete graph K_{12}?

An.. Degree → $12-1 \Rightarrow 11$ $\quad n-1$

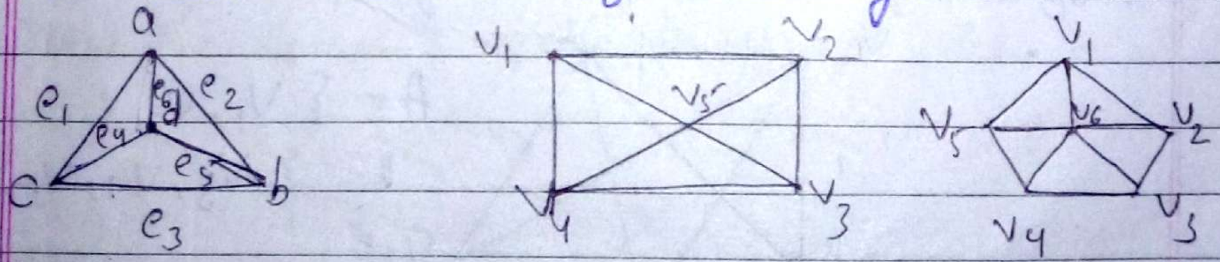
✱ Cycle →

• Continuous chain of vertices and edges start and end point is the same vertex is called cycle. But it is without of loop or parallel edges.



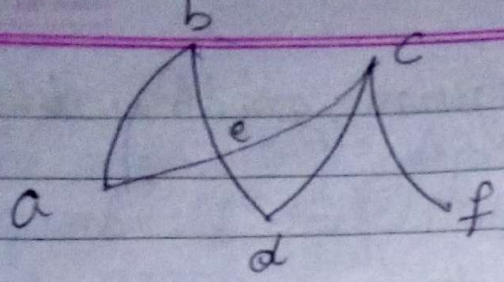
✱ Wheel →

To find wheel introduce to a new vertex in cycle and connected this new vertex to each vertex of the cycle.



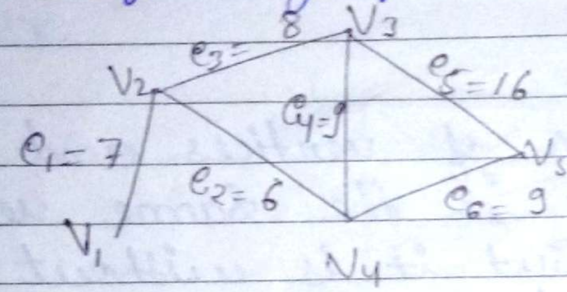
✱ Digraph →

Digraph is a picture of a graph in which each vertex is denoted by a point and each edge represent by an arc.



✪ Weighted graph →

If every edge of the graph is assigned by an integer number then the graph is called weighted graph.



✪ Bipartite graph →

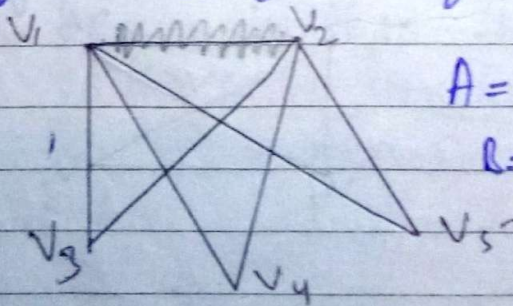
If the set of vertices can be divided into 2 non-empty subsets V_1, V_2 such that —

(i) every vertex of V_1 or V_2 is connected with at least 1 vertex of V_2 or V_1.

- So that no vertex is isolated vertex.

(ii) There is no connection betw. the vertices of V_1 of itself & V_2 of itself.

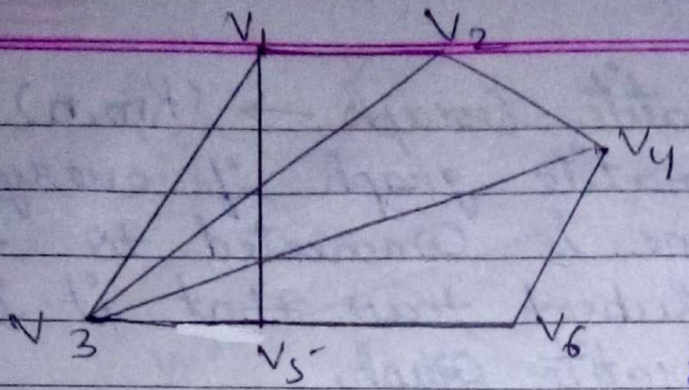
A/c def^n



$A = \{V_1, V_2\}$

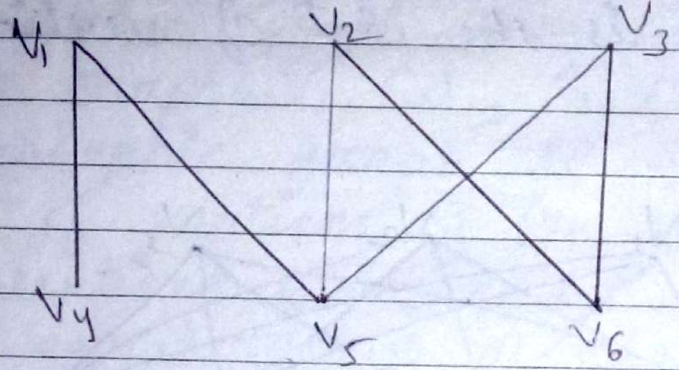
$B = \{V_3, V_4, V_5\}$

Ques → Find which of the following graphs are bipartite graph or not?



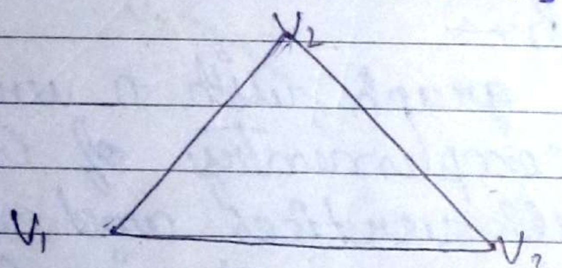
{V1, V2, V3, V4, V5, V6}

A = {V1, V2}
B = {V3, V4, V5, V6}



b/w

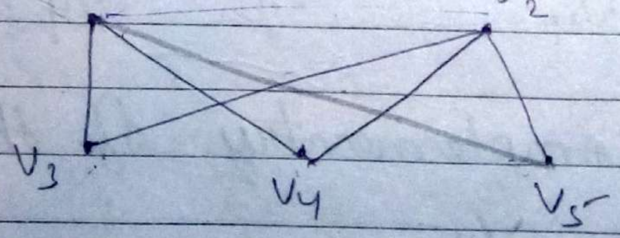
A = {V1, V2, V3}
B = {V4, V5, V6}



Not a bipartite graph

A = {V1}
B = {V2, V3}

Draw the Complete biparatite graph K2, 3

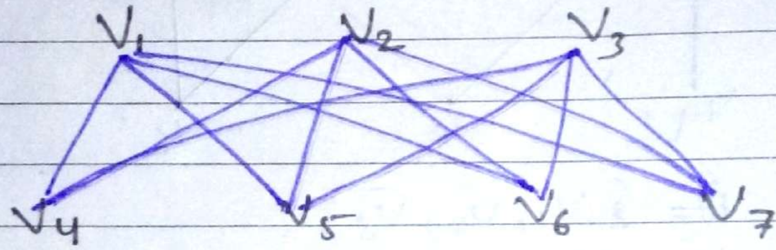


A = {V1, V2}
B = {V3, V4, V5}

Complete Biparatite Graph → (Km,n)

Complete biparatite graph if every vertex of Subset one is connected to every vertex of subset two that it is called complete biparatite graph.
It is denoted by Km,n.
Where m is the set of vertices in Subset one and n is the set of vertices of Subset two

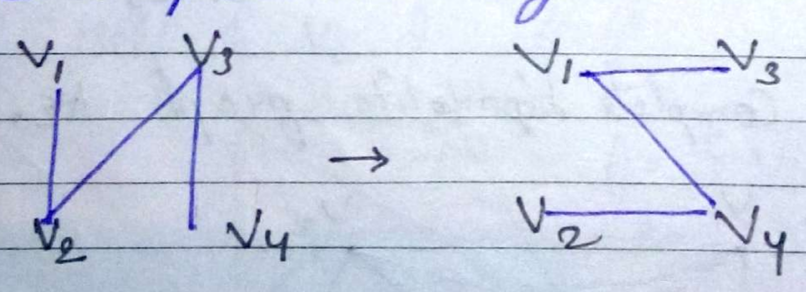
Ex → $K_{3,4}$



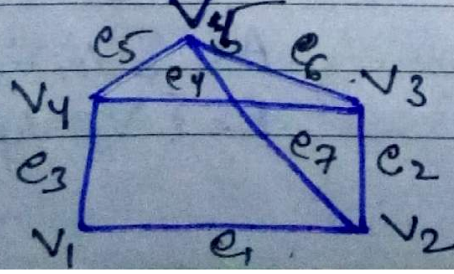
Complementry graph →

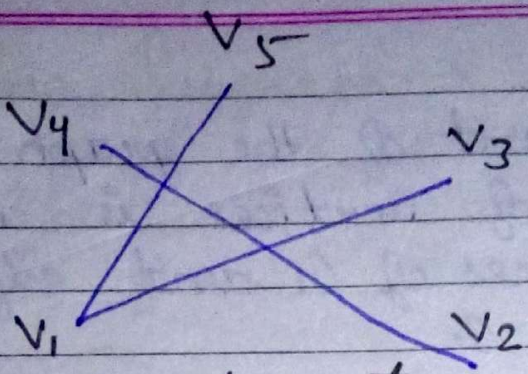
Let G(V, E) is a graph with n vertices and E edges then complementry of G is that graph of all vertices and all those edges which are not in G.
It is represented by $\bar{G}(V, E')$

Ex →



Ques → find the complementry of the graph G?



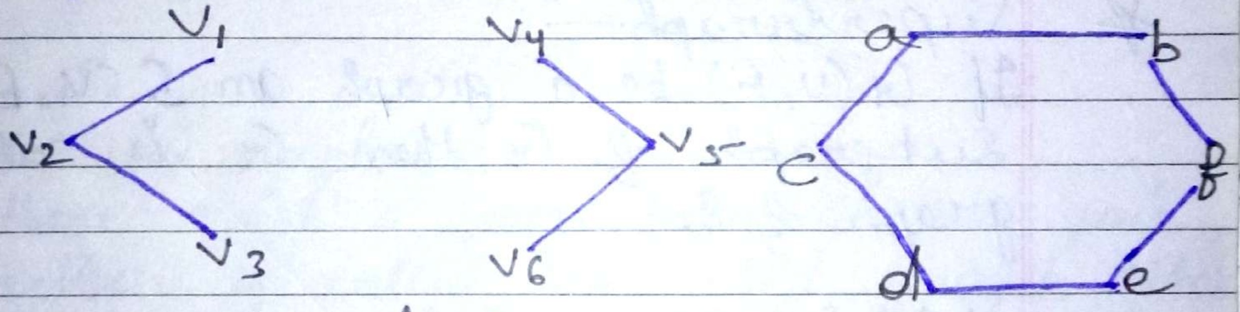


Complementry graph

✱ Isomorphic Graph →

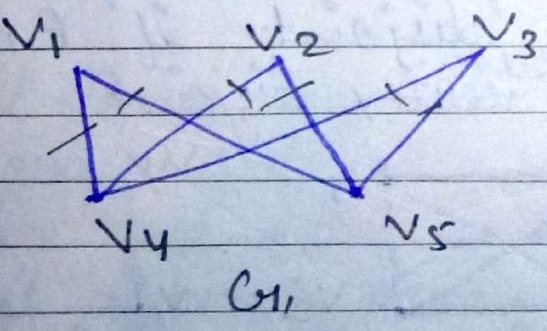
Two graphs $G_1(V_1, E_1)$, $G_2(V_2, E_2)$ are called isomorphic graph to each other if set of vertices of G_1 is equal to set of vertices of G_2 & No. of edges of G_1 is equal to No. of edges of G_1.

Ex →

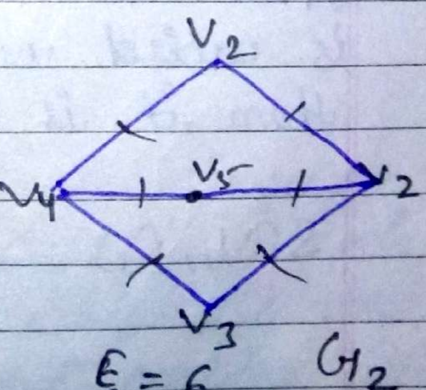


Graph - 1

Ques → Prove the following graphs are isomorphic



G_1



G_2

$E_1 = 6$ $E_2 = 6$

$V_1 = 5$ $V_2 = 5$

So, They are isomorphic graph

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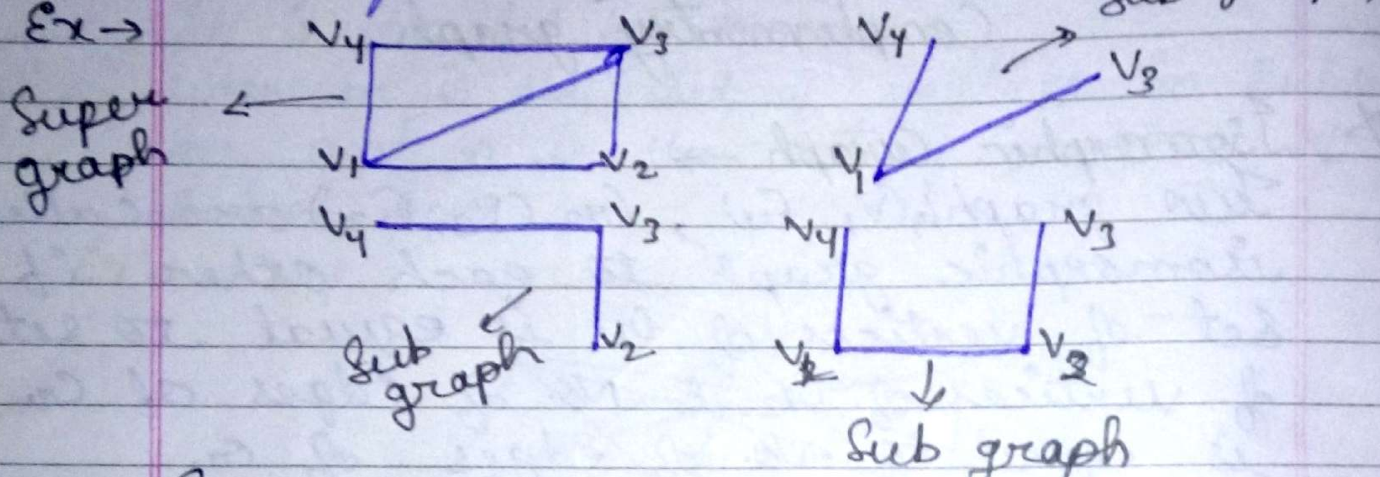
* **Subgraph →**
It is a part of the graph of G in which set of vertices is a subset of set of vertices of G and edges are also taken from G.

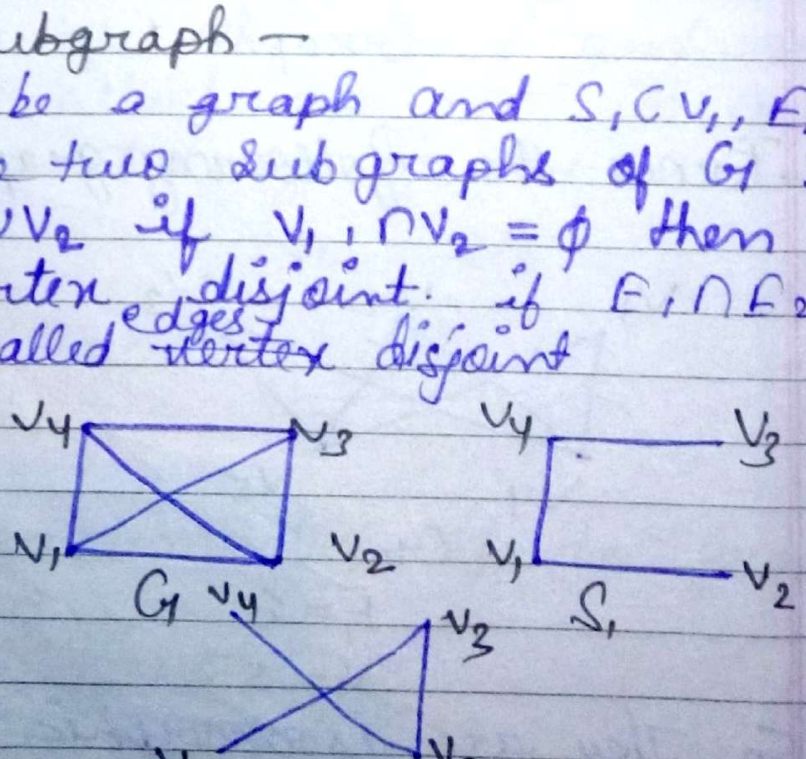
Ex →



* **SuperGraph —**
If G (V, E) be a graph an S (V₁, E₁) be Subgraph of G then G is called Super graph of S.

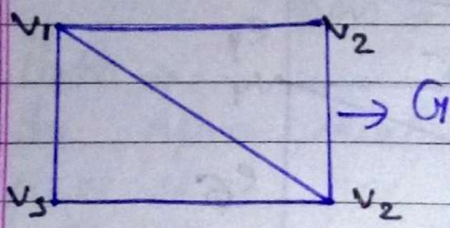
* **Disjoint Subgraph —**
Let G (V, E) be a graph and $S_1(V_1, E_1)$ & $S_2(V_2, E_2)$ be two subgraphs of G such that $V = V_1 \cup V_2$ if $V_1 \cap V_2 = \phi$ then it is called vertex disjoint. if $E_1 \cap E_2 = \phi$ then it is called ~~vertex~~ edges disjoint

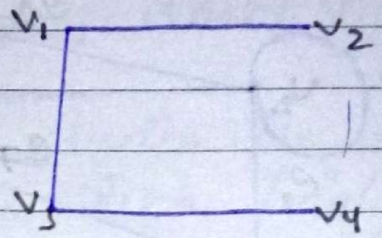
S (V₁, E₁)



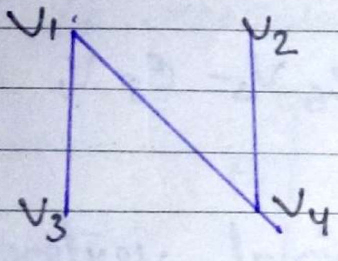
Spanning Subgraph →

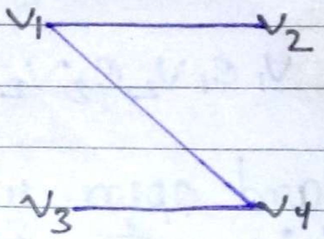
- In this graph we contain all the vertex of G without circuit is called Spanning Subgraph of G.





S_1

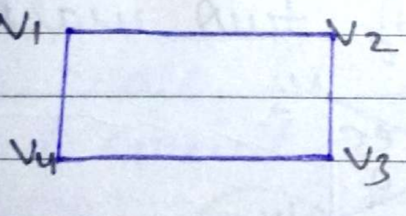




S_2 S_3

Connected graph →

- If there exist a path between every pair of vertices is called Connected graph other wise this called disconnected graph.

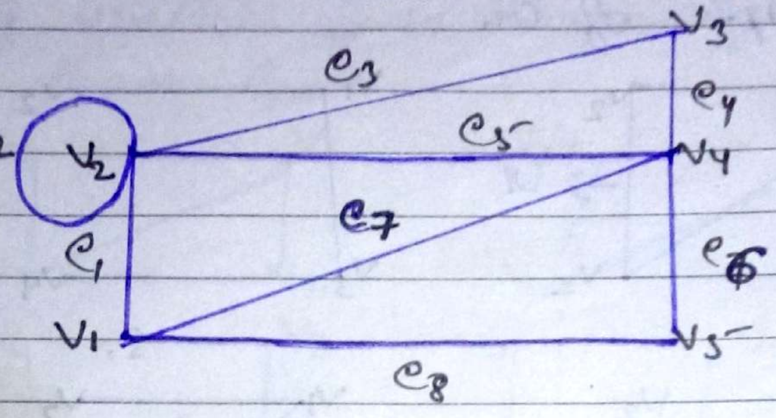


Walk Graph →

- A graph and alternate finite sequence of vertex and edges is called walk graph
- It is denoted by W in which starting vertex is called origin and ending vertex is called terminal vertex.
- It is necessary that it has only one edge between pair of vertices.

The length of walk is number of edges in walk.

Ex → Write the walk of the following graph →

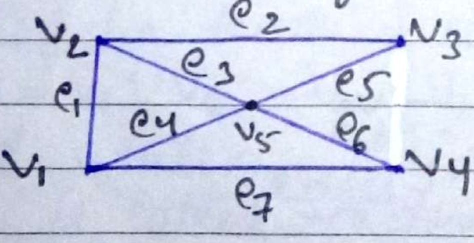


Sol^n → $w_1 = v_1 e_1 v_2 e_2 v_2 e_5 v_4 e_6 v_5 e_8 v_1$

→ Closed and open walk –

- If origin vertex and terminal vertex is the same point then that walk is called closed walk. & if origin and terminal vertex is not same point then it is called open walk. (length of walk = No. of edges).

Ques → Write the length of two walks?



Sol^n → $w_1 = v_1 e_1 v_2 e_2 v_3 e_5 v_5 e_4 v_1$

length of ~~edge~~ walk → 4

$w_2 = v_1 e_4 v_5 e_6 v_4 e_7 v_1$

length of ~~edge~~ walk → 3

$w_1 = v_1 e_4 v_5 e_6 v_4$

length of walk = 2

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Trial Graph →

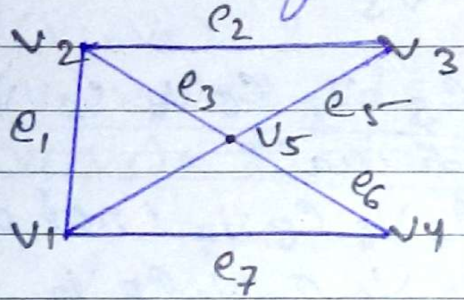
- If in walk only vertex is repeated and edges are not repeated that walk is called trial graph. It can be close & open.

open → $V_1 \neq V_1$ closed $V_1 = V_1$

Path →

- In open trial if vertex is repeated not more than one time such trial is called path. In this all the vertex should be included.
- It is represented by P.

Example →



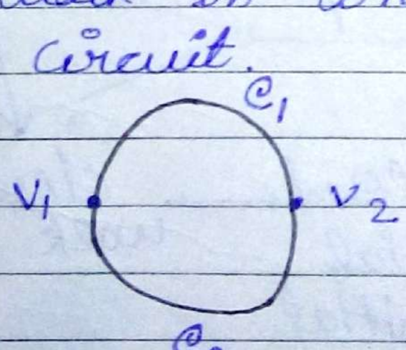
$P = V_1 e_1 V_2 e_2 V_3 e_5 V_5 e_6 V_4$

$W = V_1 e_1 V_2 e_3 V_5 e_6 V_4$, $T = V_1 e_1 V_2 e_3 V_5 e_6 V_4 e_7 V_1$

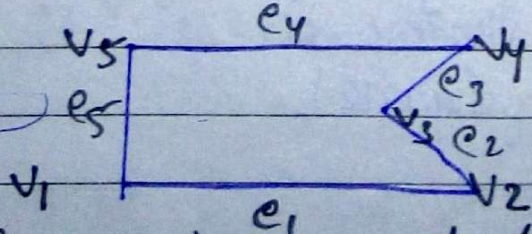
walk cannot be a path

Circuit →

- A close walk in which no edge repeated is called circuit.



$V_1 e_1 V_2 e_2 V_1$



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In This diagram is closed trial and circuit

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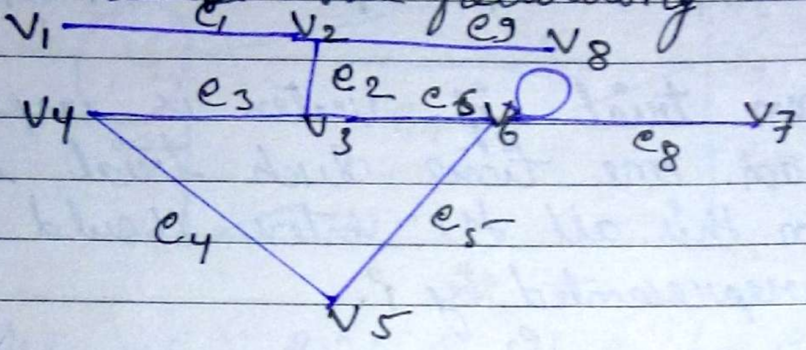
Cycles →

- A close walk in which niether vertex nor edges are repeated only initial and ~~origin~~ or terminal vertex can be same.

Ques→ Characterise the following walks —



$W_1 = v_1\, e_1\, v_2\, e_2\, v_3\, e_6\, v_6\, e_7\, v_6\, e_5\, v_5\, e_4\, v_4$
open walk

$W_2 = v_5\, e_5\, v_6\, e_7\, v_6\, e_8\, v_7$ (not path/walk)

$W_3 = v_1\, e_1\, v_2\, e_2\, v_3\, e_6\, v_6\, e_8\, v_7$

$W_4 = v_1\, e_1\, v_2\, e_9\, v_8\, v_4$
∴ ~~Now~~ None of this

Ques- Relation between Graph, Sub Graph, walk, path, circuit.

Graph → Sub Graph → walk → open walk (initial & terminal vertex diff) → open trial (vertex repeated only) → (vertex not repeated & edges not repeated) Open path

walk → closed walk → closed trial (initial & terminal vertex is same) → closed path (no edge & no vertex repeated)

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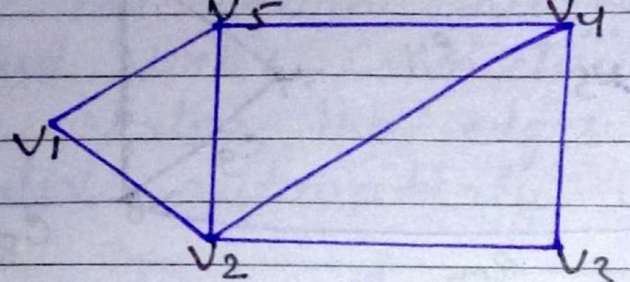
✱ Distance between vertices →

The length of the shortest path of two vertices of connected graph is called distance of vertices.



$d(V_1, V_2) = 1$

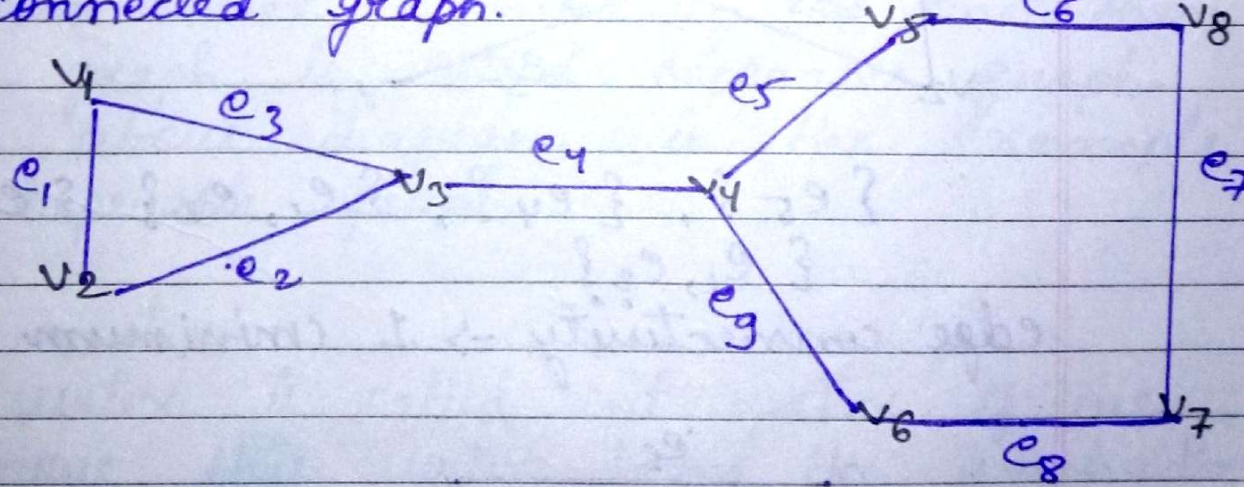
$d(V_1, V_3) = 2$

$d(V_1, V_4) = 2, \; d(V_1, V_4) = 3$

✗ $\boxed{d(V_1, V_4) = 2, \; d(V_1, V_4) = 3}$ ✗

✱ Bridge →

An edge which is used for connecting two graphs. If we remove this edge then the connected graph is convert into disconnected graph.



In this diagram e_4 work as a bridge because it is connecting two graphs.

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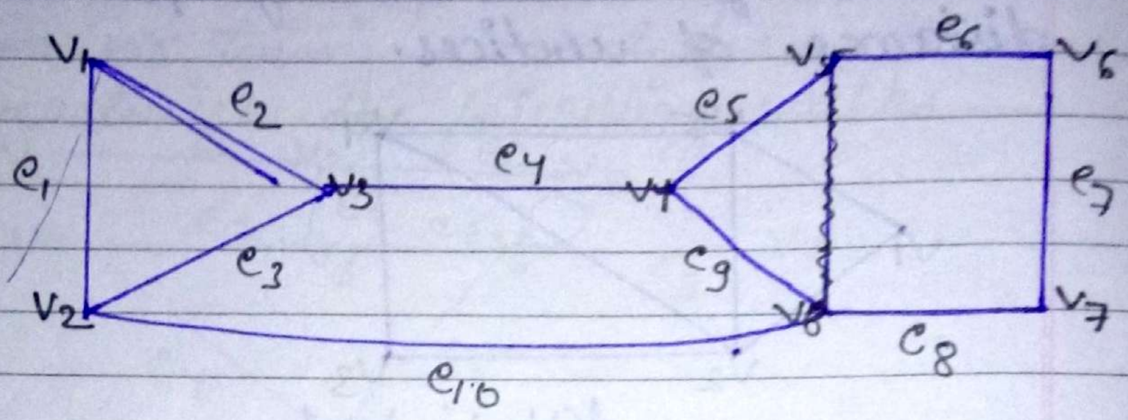
Cut - Set →

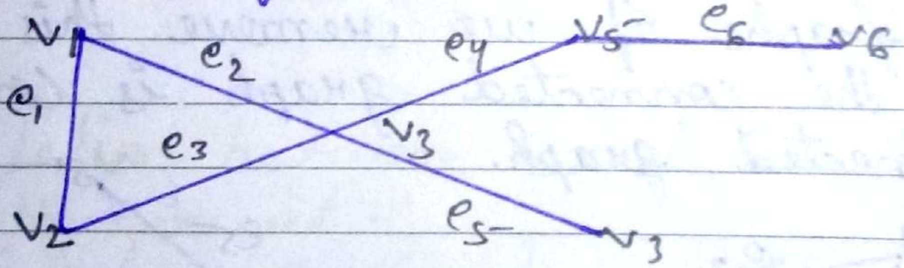
A set of edges (Bridge) is called cut set



Cut set → $A = \{e_4, e_{10}\}$

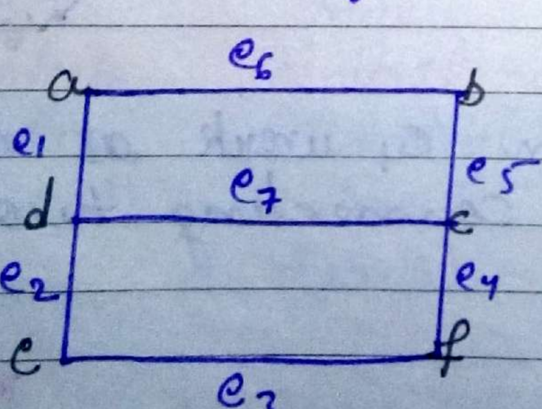
Edges Connectivity →

Cut sets of the graph in which we have minimum number of edges is called edge ~~contino~~ connectivity.



$\{e_5\}, \{e_4\}, \{e_1, e_2\}$ $\{e_6\}$
$\{e_1, e_2\}$

edge connectivity ⇒ 1 (minimum no of edges)

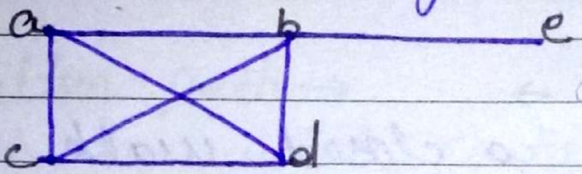


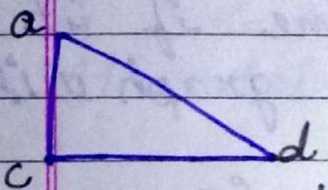
$\{e_1, e_6\}$ $\{e_2, e_4\}$ $\{e_1, e_7\}$ $\{e_2, e_3\}$

edge connectivity $\Rightarrow$ 2

✱ Vertex connectivity →

The minimum number of vertex is called vertex connectivity if we remove the vertex the edges incident to that vertex automatically remove.





remove a & b

remove b

Then, the vertex connectivity is 1

✱ Seperable Graph →

- If the vertex connectivity is one then the graph is called seperable graph. The above diagram is the example of seperable graph.

✱ Cut vertex →

A vertex is called cut vertex if we remove this vertex from the graph then the graph is convert into seperable graph

Note →

$$\boxed{\text{vertex connectivity} = 2e/n}$$

$e \rightarrow$ no. of edge

$n \rightarrow$ no. of vertex

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Ques → find the maximum vertex connectivity having vertex are 8 and edges are 16.

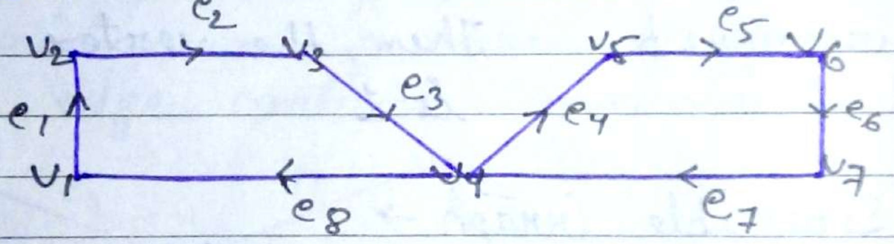
Solution → formula → $\frac{2e}{n}$

$\Rightarrow \frac{2 \times \cancel{16}^{2}}{\cancel{8}}$

Vertex connectivity ⇒ 4 Ans

✡ Eular line →

Eular line a closed walk of connected graph is called Eular line if it contain all the edges of graph without repetation



=> $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5 e_5 v_6 e_7 v_7 e_7 v_4 e_8 v_1$

✡ Euler graph →

A graph in which Euler line exist is called eular graph.

✡ Unicursal line →

An open walk which contain all the edges of G without repetation is called unicursal line.

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Eular formula → $v - e + r = 2$

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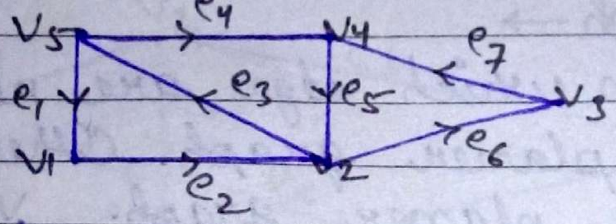
✯ Unicursal graph or open Eular line →

A graph which contain unicursal line is called unicursal graph.

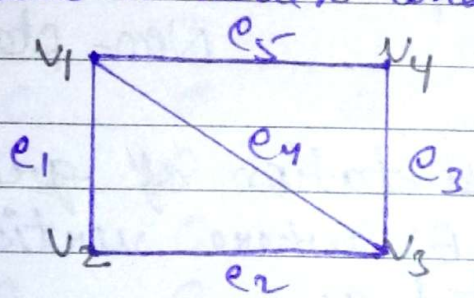


$W = V_5\, e_1\, V_1\, e_2\, V_2\, e_3\, V_5\, e_4\, V_4\, e_5\, V_2\, e_6\, V_3\, e_7\, V_4$

✯ Hamilton path →

A path in which contain all the vertex of G is called Hamilton path.

It contains n vertex and (n−1) edges



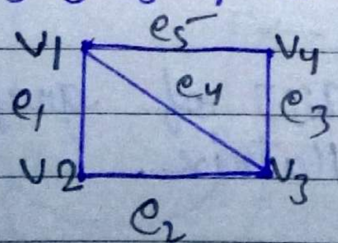
$V_1\, e_1\, V_2\, e_2\, V_3\, e_3\, V_4$

✯ Hamilton Circuit →

If in the Hamilton path origin & terminal vertex is same then it is called Hamilton circuit.

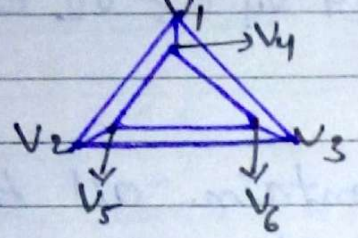
It contain n vertex and n edges.

$V_1\, e_1\, V_2\, e_3\, V_3\, e_3\, V_4\, e_5\, V_1$

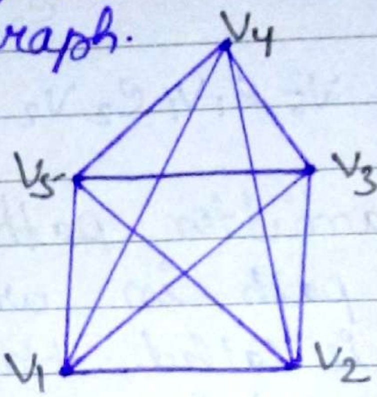


* Hamilton graph →
A graph in which hamilton path exist.

* Planer graph →
A graph in which edges are not intersect is called planer graph. Otherwise, it is called non-planer graph.



Planar Graph



Non-planar Graph

* Matrix representation of graph →
A graph G(V, E) where vertices & edges are represented as –

$V = \{v_1, v_2, v_3 \ldots\ldots v_n\}$

$E = \{e_1, e_2, e_3 \ldots\ldots e_n\}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} \end{bmatrix}$$

* Incident Matrix →
A matrix $A = [a_{ij}]$ of graph G(V, E) is called Incident Matrix.

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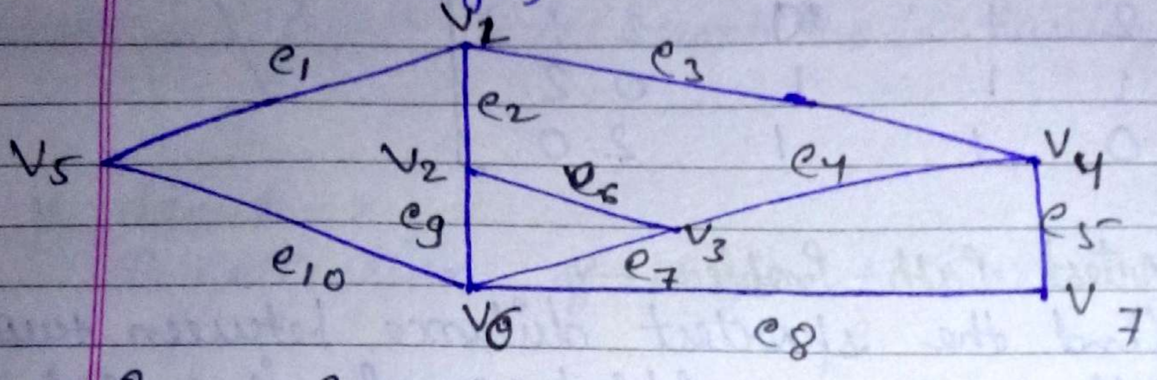
$[a_{ij}] = \begin{cases} 1 & \text{if } v_i \text{ vertex connected to } e_j \text{ edge} \\ 0 & \text{if } v_i \text{ vertex not connected to } e_j \text{ edge} \end{cases}$

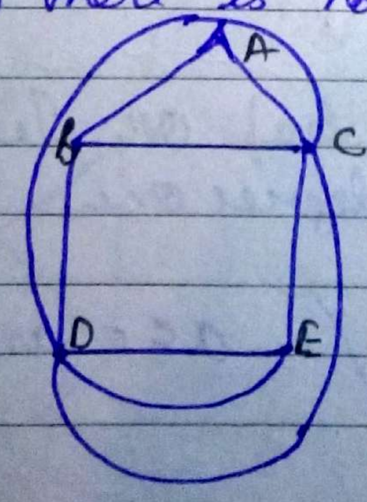


$$
\begin{array}{c|cccccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \\
\hline
V_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
V_2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
V_3 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
V_4 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
V_5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
V_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
V_7 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
V_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

* Adjancy Matrix

A matrix is $A = [a_{ij}]$ of graph $G(V, E)$ is called Adjancy Matrix. where,

$a_{ij} = \begin{cases} K & \text{No. of edges between } V_i \text{ \& } V_j \\ 0 & \text{there is no edges between } V_i \text{ \& } V_j \end{cases}$



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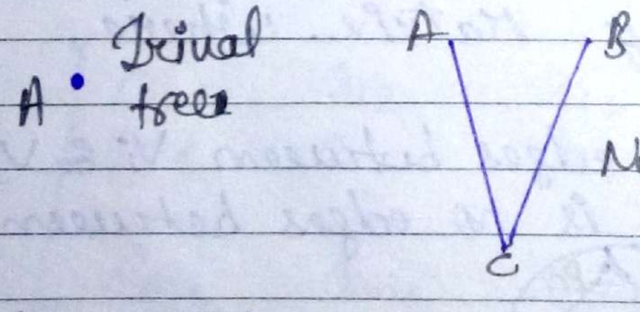
$$
A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix}
0 & 1 & 2 & 1 & 0 \\
1 & 0 & 1 & 1 & 1 \\
2 & 1 & 0 & 2 & 1 \\
1 & 1 & 1 & 0 & 2 \\
0 & 1 & 1 & 2 & 0
\end{bmatrix}
\end{array}
$$

* **Shortest Path Problem →**
To find the shortest ditance between two vertices of a weighted graph is called Shortest path problem.

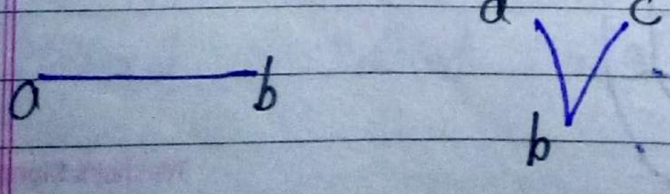
* **TREE →** A connected undirected graph without any loop of parallel edges is called tree.

* **Trival Tree (degenerate) →**
A tree having single vertex is called trival tree otherwise it is called non-trival tree.



Non trival tree

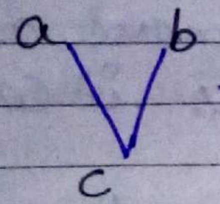
* **Pendent Vertex or Leaf or Terminal Node →**
A vertex having degree one



a & c are leaf

* Branch Node →

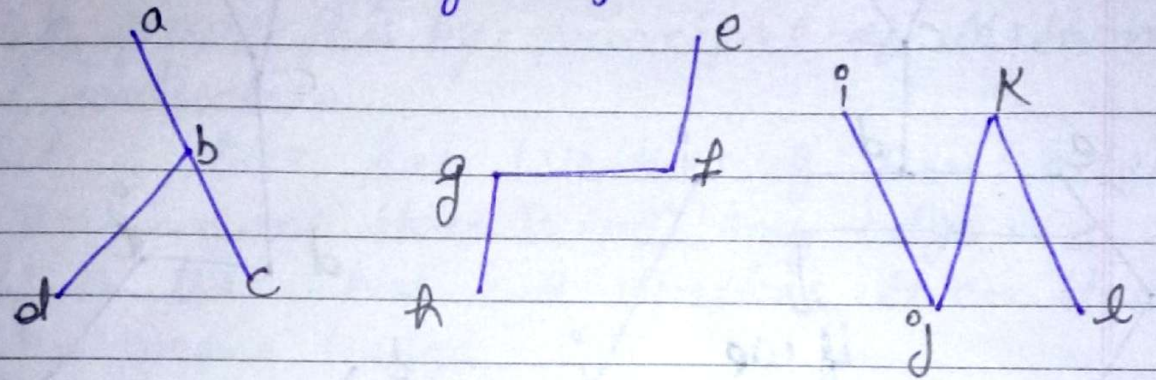
A vertex having more than one degree



b is branch node having 2 degree

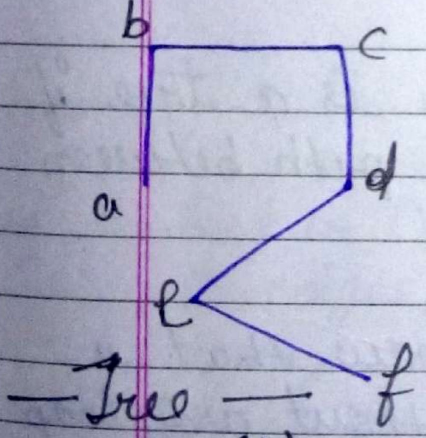
* forest →

A collection of disjoint trees.

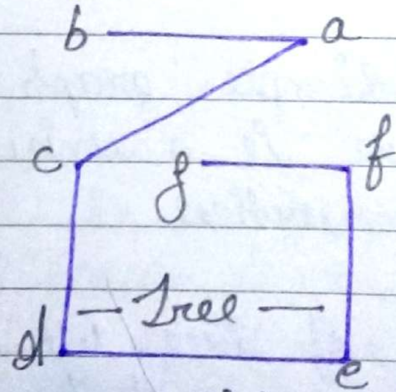


Ques. which of the following graphs are tree —

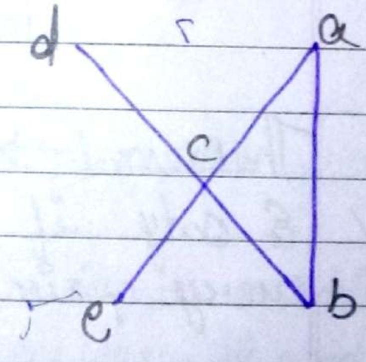
Tree → The open graph and never closed figure. In tree vertex is more than one edge.



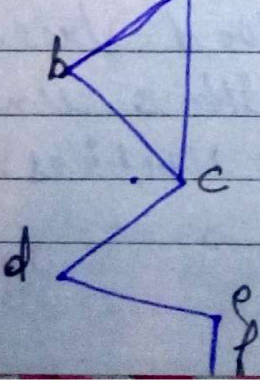
(1)



(2)



(3)



(4)

fig. (1) & (2) are tree

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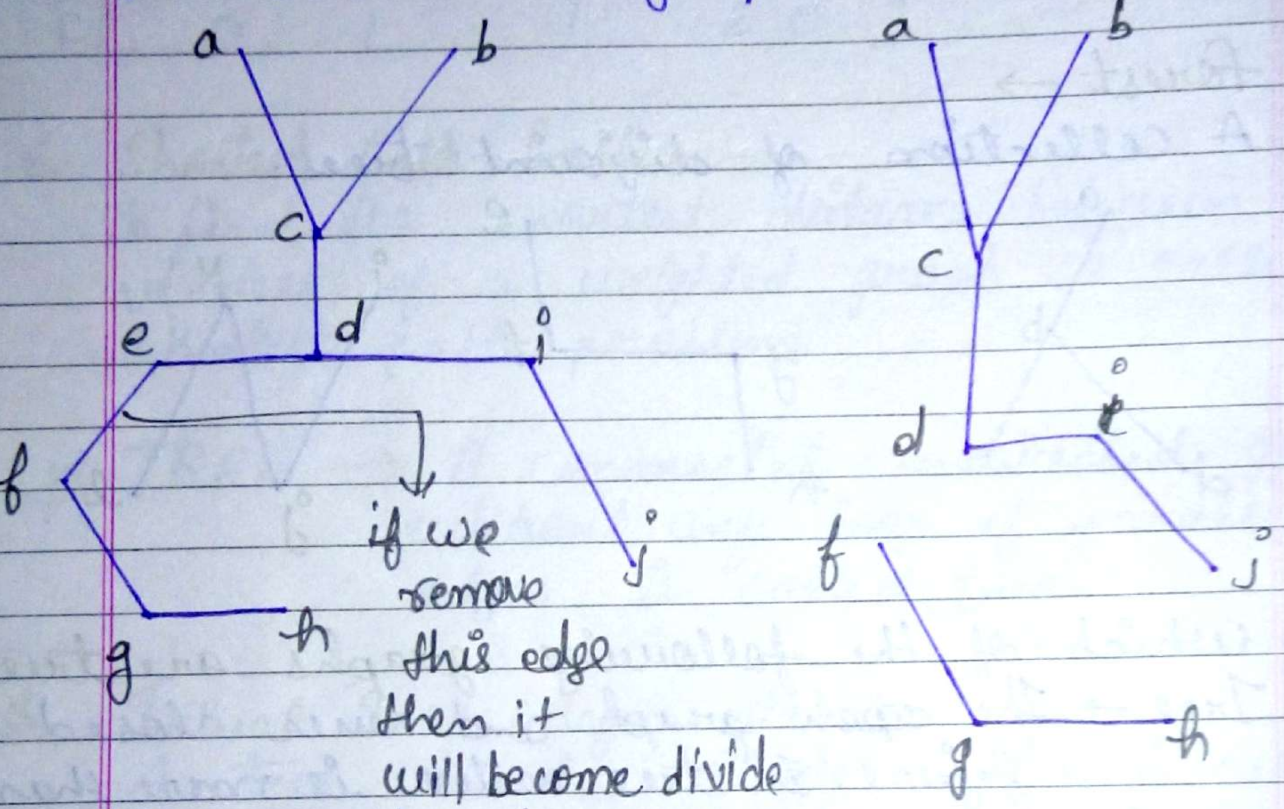
Minimal Connected Graph →

A connected graph is said to minimal connected graph if we remove any edge between two vertices then it converted into disconnected graph.



if we remove this edge then it will become divide into two trees

Theorem 1 → A simple graph G is a tree if & only if there is a unique path between every pair of vertices.

prove: By defination of tree. we know that a graph which is connected without any loop or parallel edges is called tree.

Graph (G) is a tree with a unique path between every pair of vertices.

Uniqueness →

Let there is

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Then there is a circuit in a tree which is Contradict our defination of tree hence ~~G has~~ has only single or unique path between pair of vertices.

\# Theorem 2 → A tree with n vertices as (n-1) edges where n is a integer number.

Prove: We prove this by principal of Mathematical induction.

- Let a tree has 1 vertices of then edges = (n-1) means there is no any edge.
- If a tree has two vertices then there is (n-1) means 1 edge.
- Let take it is true for n∈K vertices.
- If a tree has K vertex then it has K-1 edges.
- If a tree has K+1 vertex then it has (K+1-1) so it is true for all N.

Ques - find the regularity r of the complete graph Kn of n vertices?

Sol^n → Let G (V, E) be a graph with n vertices & E edges, since G is r regular hence every vertex of it has degree 'r'. —(1)
So the sum of degree of all vertex = nr
Similarly G(V, E) is also complete graph with n vertices, so every vertex is connected with remaining (n-1) vertices.
Hence, degree of each vertex be = (n-1)
Sum of the degree of all vertex = n(n-1) —(2)

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from eq^n (1) & (2)

$$nr = n(n-1)$$

So, $r = n-1$

Hence for a complete graph K_n with n vertex, the regularity of graph is $n-1$.

formula → $\text{Regularity} = n - 1$

n = No. of vertex

Ques ⇒ Show that in Biparatite graph with n vertices the total no. of edges not exceed. $n^2/4$

Sol- V_1 and V_2 G

$V_1 \to \{p\}$

$V_2 \to \{q\}$

$$A = \{v_1, v_2, v_3 \ldots p\}$$

$$B = \{v_{p+1}, v_{p+2}, v_{p+3} \ldots q\}$$

$$p + q = n \quad (1) \qquad pq = m \quad (2)$$

$$p = q$$

(all are subset) $q + q = n$

$$2q = n$$

$$q = \frac{n}{2} \quad (3)$$

$$p + \frac{n}{2} = n$$

$$p = \frac{n}{2} \quad (4)$$

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from eq^n (2) (3) & (4)

$$m = \frac{n^2}{4}$$

max. no. of edges

Theoram - 3

Q.2 A graph G with 'n' vertices n-1 edges and no circuit is connected or tree.

Prove:- Let G be a circuit less graph with n vertices & (n-1) edges suppose. G is disconnected graph in this case G will contain two and more than two circuit

For example - let G has v_1 v_2 vertices than G_1 is a graph with v_1 vertex and G_2 is a graph with v_2 vertex and there is no connection b/w G_1 and G_2 than G_1 is a disjointed graph if we joint v_1 and v_2 vertex than $G_1 \cup G_2$ at it become connected graph with two vertices and one edges hence, our assumption is wrong than G is disconnected graph ⇒ G is connected graph.

Theoram - 4

If graph is a tree if and if It is minimally connected.

Let G be a minimally connected graph than by defination of minimally connected graph we know that it has it does not contain any parallel edges or circuit and by elominating and n edges b/w with two vertex from the graph

is contain disconnected graph.
hence. G is a tree conversally let G is a tree
now we have to prove that G is a minimally
connected graph. by deffination of the tree
we know that G has does not contain parallel
edges or circuit
Let G is a also not minimally graph than
by elementing any edges of the graph be it
will still connected graph which contract
over defⁿ of the tree.
hence. So over assumption is wrong G is a not
connected graph.