SHREE
DATE: / /
PAGE NO.:

12/11/13

Chapter -
Logic and Proofs

✱ Propositions →

- A proposition is a declarative sentence which is either true or false, but cannot be both true and false at the same time.
- The sentence must not convey a situation which is imperative, Interrogative, exclamatory.

→ Examples –

(i) Jaipur is in Rajasthan. → Truth
(ii) Do you speak French? → Interrogative, not a Statement
(iii) Do your homework. → Imperative, not a statement
(iv) $8 < 7$. False

✱ Compound propositions →

Propositions that can be broken down into simpler propositions are called compound propositions.

Example →

• (a) It is raining and I am not going outside
(It is compound proposition because it is composed of two sentences).
(i) It is raining.
(ii) I am not going outside.
(b) It is raining or I am not going outside.
It is also a compound statement.

Teacher's Signature

DATE: / /
PAGE NO.:

* Logical connectives –

- Symbols used to combine two or more simple propositions to form a compound proposition is called connectives or logical connectives

S.NO.	Name of the connective	Connective word	Symbol	used as	Read as
1.	Conjuction	and	$\wedge$	$p \wedge q$	p and q
2.	Disjunction	or	$\vee$	$p \vee q$	p or q
3.	Negation	not	$\sim$	$\sim p$	not p
4.	conditional	If..... then	$\Rightarrow$	$p \Rightarrow q$	p implies q
5.	Biconditional	If and only if (iff)	$\Leftrightarrow$	$p \Leftrightarrow$	p implies q and q implies p

* Conjuction, $p \wedge q \rightarrow$

- Any two propositions can be combined by the word 'and' to form a compound proposition called the conjuction of the two propositions.

Truth table for conjuction, $p \wedge q \rightarrow$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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DATE: / /
PAGE NO.:

Example → P and q are statement variables then if P is false and q is true then $P \wedge q$ is
(a) false
(b) True
(c) True or false
(d) True or false but not both

Solution → If P is false and q is true, the $P \wedge q$ will be false.

* Disjunction, $P \vee q$

- Any two propositions can be determined by the word "or" to form a compound proposition called the disjunction of the two propositions.

Truth table

P	q	$P \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example →
(i) Delhi is in U.P., or 2+2=4. True
(ii) Delhi is in U.P, or 2+2=5. false
(iii) Delhi is in Rajasthan, or 2+2=4. True
(iv) Delhi is in Rajasthan, or 2+2=5. False

* Negation, $\sim P$

- P is a statement, the negation of P is the statement "not P."

Teacher's Signature

DATE: / /
PAGE NO.:

→ Truth table –

p	$\sim p$
T	F
F	T

Example → If p is "Jaipur is in ~~not~~ Rajasthan", then $\sim p$ will be "It is not the case that jaipur is in Rajasthan" or simply "Jaipur is not in Rajasthan".

Example → Determine the truth value of the following statements if p is true and q is false:

(a) $\sim p \wedge q$

(b) $\sim p \vee \sim q$

Solution → We frame the following truth table

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If p and q are statements, the compound statement "if p then q" is called conditional statement or implification.

Truth table of $p \Rightarrow q$ in terms of p and q

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Date: / /

Conditional Connective →

- If P and q are statements, the compound statement "if P then q" is called a conditional statement or implication.
- It is denoted by $P \rightarrow q$ or $P \Rightarrow q$ and read as "P implies q".
- $P \longrightarrow$ hypothesis
- $q \longrightarrow$ conclusion

Truth Table

P	q	$P \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

→ "$P \Rightarrow q$"

Biconditional Connective →

If P and q are statements, the compound statement : P if and only if q, denoted by $P \Leftrightarrow q$ is called an equivalence or biconditional.

The connective "if and only if" is denoted by symbol $\Leftrightarrow$

Truth table

P	q	$P \Leftrightarrow$
T	T	T
T	F	F
F	T	F
F	F	T

"$P \Leftrightarrow q$"

Page No.:
Date: / /

* De & Morgan's law →

The following are known as De Morgan's laws for statements.

i) $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

ii) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

Prove (i) → $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

L.H.S →

p	q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

R.H.S →

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Then, by this We can say L.H.S = R.H.S.
De Morgan's first law prove

(ii) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

L.HS →

p	q	$p \vee q$	$\sim(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

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R.H.S →

p	q	~p	~q	~p ∧ ~q
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Hence, L.H.S = R.H.S

So, We can say that second law of De'morgan's has proved

Ex → Compute the truth table of the statement

$(p \to q) \leftrightarrow (\sim q \to \sim p)$

Solution

p	q	p → q	~q	~p	~q → ~p	(p→q) ↔ (~q → ~p)
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T