Number System

✱ Types of Number System →

(1) Binary (Base = 2)
(2) Decimal (Base = 10)
(3) Octal (8)
(4) Hexadecimal (9 to A, B, ...)

✱ Conversion Binary to decimal form

(i) $(101011)_2$

$\Rightarrow 1\times2^5 + 0\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$\Rightarrow 43$

(ii) $(101.011)_2$

$\Rightarrow 1\times2^2 + 0\times2^1 + 1\times2^0 + 0\times2^{-1} + 1\times2^{-2} + 1\times2^{-3}$

$\Rightarrow 4 + 0 + 1 + 0 + \frac{1}{4} + \frac{1}{8}$

$\Rightarrow \cancel{\frac{32+16+8+0+2+1}{8}}$

$\Rightarrow 5\ \cancel{7} + 0.25 + 0.125$

$\Rightarrow 7.375$

✱ Conversion decimal to Binary →

(i) $(96)_{10}$

2	96	Remainder
2	48	0
2	24	0
2	12	0
2	6	0
2	3	0
	1	1

$\rightarrow (1100000)_2$

(iii) $(0.8175)_{10}$

$0.8175 \times 2 = 1.6350 = 1$

$.6350 \times 2 = 1.2700 = 1$

$.2700 \times 2 = 0.5400 = 0$

$.5400 \times 2 = 1.0800 = 1$

$0.0800 \times 2 = 0.1600 = 0$

No. → $(0.1101)_2$

(iv) $(0.85)_{10}$

$0.85 \times 2 = 1.70 = 1$

$0.70 \times 2 = 1.40 = 1$

$0.40 \times 2 = 0.80 = 0$

$0.80 \times 2 = 1.60 = 1$

~~0.1101~~ $0.60 \times 2 = 1.20 = 1$

$0.20 \times 2 = 0.40 = 0$

$0.40 \times 2 = 0.80 = 0$

No → $(0.1101001)_2$ $\quad 0.80 \times 2 = 1.60 = 1$

(v) $(\underset{\text{divide}}{13}.\underset{\text{multiply}}{875})_{10}$

		Remainder
2	13	
2	6	1
2	3	0
	1	1

(1101

$0.875 \times 2 = 1.650 = 1$

$0.650 \times 2 = 1.300 = 1$

$0.300 \times 2 = 0.600 = 0$

$0.600 \times 2 = 1.200 = 1$

$0.200 \times 2 = 0.400$

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(vi) $(21.6875)_{10}$

		Remainder
2	21	1
2	10	0
2	5	1
2	2	0
	1	

NO. ⇒ 10101.1011

$0.6875 \times 2 = \cancel{1}.3750 = 1$

$0.3750 \times 2 = \cancel{0}.7500 = 0$

$0.7500 \times 2 = \cancel{1}.5000 = 1$

$0.5000 \times 2 = \cancel{1}.0000 = 1$

Conversion decimal to octal →

(i) $(726)_{10}$

		Remainder $(1326)_8$
8	726	6
8	90	2
8	11	3
	1	

(ii) (922.296875)

		R
8	922	2
8	115	3
8	14	6
	1	

(1632

$0.296875 \times 8 =$

Conversion octal to decimal →

(i) $(522)_8$

$5 \times 8^2 + 8^1 \times 2 + 2 \times 8^0$

$\Rightarrow 64 \times 5 + 16 + 2$

$\Rightarrow 320 + 16 + 2$

No. $\Rightarrow (338)_{10}$

320
16
2
33

(ii) $(1632.23)_8$

$1 \times 8^3 + 6 \times 8^2 + 3 \times 8^1 + 2 \times 8^0 + 2 \times 8^{-1} + 3 \times 8^{-2}$

$\Rightarrow 512 + 24 + 2 + \frac{2}{8} + 3 \times \frac{1}{64}$

$\Rightarrow 512 + 24 + 2 + \frac{1}{4} + \frac{3}{64}$

$\Rightarrow 922 + 0.25 + 0.0468$

$\Rightarrow (922.2988)_{10}$

* Conversion of hexa decimal to decimal →

(i) $(230)_{16}$

$\Rightarrow 2 \times 16^2 + 3 \times 16^1 + 0 \times 16^0$

$\Rightarrow 612 + 48 + 0$

$\Rightarrow (560)_{10}$

A to F

(ii) $(3A4E)_{16}$

or $(3\,10\,4\,14)$

$\Rightarrow 3 \times 16^3 + A \times 16^2 + 4 \times 16^1 + E \times 16^0$

$\Rightarrow 12288 + 2560 + 64 + 14$

$\Rightarrow (14926)_{10}$

(iii) $(2AF.A)_{16}$

$\Rightarrow 2\times16^2 + A\times16^1 + F\times16^0 + A\times16^{-1}$

$\Rightarrow 2\times256 + 10\times16 + 15\times1 + 10\times16^{-1}$

$\Rightarrow 512 + 160 + 15 + \cancel{10}^{5}\times\frac{1}{\cancel{16}\,8}$

$\Rightarrow 512 + 160 + 15 + \frac{5}{8}$

$\Rightarrow 512+160+15+0.625$

$\Rightarrow 687.625$

Conversion of decimal to Hexadecimal →

i) $(7295)_{10}$

16	7295	R
16	455	15 = F
16	28	7
16	1	12 = C

No. ⇒ $(1C7F)_{16}$

ii) $(1046.25)_{10}$

⇒

16	1046	Remainder
16	65	6
	4	1

No. ⇒ $(416.4)_{16}$

~~2×16¹ + 5×16⁰ ⇒~~

$0.25\times16 = 4.00 = 4$

iii) $(73.625)_{10}$

16	73	Remainder
	4	9

⇒ $(49.10)_{16}$

⇒ $(49.A)_{16}$

$0.625\times16 \Rightarrow 10.000 = 10$

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Conversion of Octal to Binary →

0 ⇒ 000
1 ⇒ 001
2 ⇒ 010
3 ⇒ 011
4 ⇒ 100
5 ⇒ 101
6 ⇒ 110
7 ⇒ 111

(i) $(45)_8 = (100101)_2$

(ii) $(67025)_8 = (110111000010101)_2$

(iii) $(13.64)_8 \Rightarrow (001011.110100)_2$

(iv) $(12.36)_8 \Rightarrow (001010.011110)_2$

(v) $(010100111101)_2 \Rightarrow (2475)_8$

(vi) $(001101.110100)_2 \Rightarrow (15.64)_8$

Binary to Hexa decimal →

Hexa	Binary		
0	⇒ 0000	9	⇒ 1001
1	⇒ 0001	A	⇒ 1010
2	⇒ 0010	B	⇒ 1011
3	⇒ 0011	C	⇒ 1100
4	⇒ 0100	D	⇒ 1101
5	⇒ 0101	E	⇒ 1110
6	⇒ 0110	F	⇒ 1111
7	⇒ 0111		
8	⇒ 1000		

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(i) $(01110011)_2 \Rightarrow (73)_{16}$
(ii) $(010110101100)_2 \Rightarrow (5AC)_{16}$
(iv) $(00101101.01011)_2 \Rightarrow (2D.58)_{16}$
(v) $(5AC)_{16} \Rightarrow (010110101100)_2$

Hexa decimal to octal form →

(i) $(1AF)_{16} \Rightarrow (000110101111)_2 \Rightarrow (0657)_8$
$\Rightarrow (657)_8$

Octal to Hexa decimal →

(i) $(32.4)_8 \Rightarrow (00011010.1000)_2 \Rightarrow (1A.8)_{16}$

Complex Numbers →

$i = \sqrt{-1}$
$\sqrt{-4} = 2i$
$\sqrt{-2} = \sqrt{2}\,i$

Addition in Binary System →

(i) $(75)_{10} + (105)_{10} = (\ \ )_2$

		R		Remainder →
2	75		2 | 105	
2	37	1	2 | 52	1
2	18	1	2 | 26	0
2	9	0	2 | 13	0
2	4	1	2 | 6	1
2	2	0	2 | 3	0
	1	0	1	1

$$
\begin{array}{r}
1001011 \\
+\ 1101001 \\
\hline
10110100
\end{array}
$$

✯ Subtraction Binary to Binary →

(i)
```
   10001
 -  1111
 -------
   00010
```

✯ Multiplication →

(i)
```
      10111
     × 110
    -------
      00000
     10111x
    10111xx
   ---------
   10001010
```

✯ Dividation →

```
101 ) 1011101 ( 1001
      101
      -----
       × 110
         101
        -----
         × 11
```

quotient = 1001, and remainder = 11.

✯ Octal to octal →

(i)
```
    6 7 4 5
 +    3 7 6
 ----------
      3 4 3
```

6+5 = 11

base → 8

3

Subtraction →

```
   2153
 -  734
 ------
   1217
```

* Multiplication →

$(732)_8 \times (46)_8$

$\Rightarrow$ 5434

3550×

43134

$(732)_8 \times (46)_8$

I product = 732×6

= 5434, [6×2=12 = 4+8 = 4, and Carry 1

6×3=18, 18+1=19 = 3+16

= 3+2×8 = 3 and Carry 2

8×7=42, 42+2= 44 = 4+40

= 4+ 5×8 = 4, and Carry 5]

II product = 732×4

= 3550, ∵ [2×4= 8=0, and Carry 1

4×3=12, 12+1=13, 5+8

$\Rightarrow$ 5+8×1 = 5, and Carry 1

7×4=28, 28+1= 29.

= 5+24 = 5+8×3

5, and Carry 3

Carry: 1

15434

+ 35500

43134

4+0 = 4

3+0 = 3

4+5 = 9 = 8+1 = 1+0 + Carry 1

~~5+5=10=10+1 =11 = 8+0+Carry 1~~

5+5=10=10+1=11 = 3+8 + Carry 1

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Addition →

$$\begin{array}{r} 1\ 1\ 1\ \ \\ 6745 \\ +\ 378 \\ \hline 7343 \\ \hline \end{array}$$

For the I column at the extreme right,

$6+5 = 11 = 3+8 = 3+0$ + carry 1

$7+4 = 11 = 11+1 = 4+0$ + carry 1

$7+3 = 10 = 10+1 = 3+0$ + carry 1

Complex Numbers –

- A number of the form of $x+iy$, where x and y are real numbers and $i=\sqrt{(-1)}$ is called a complex number.

$$z = \underbrace{x}_{\text{Real part}} + \underbrace{iy}_{\text{Imaginary part}}$$

Example → $\sqrt{-4} \Rightarrow \sqrt{4\times(-1)} \Rightarrow 2i$

$\sqrt{-2} \Rightarrow \sqrt{2\times(-1)} \Rightarrow \sqrt{2}\,i$

$\sqrt{-12} \Rightarrow \sqrt{4\times 3\times(-1)} \Rightarrow 2i\sqrt{3}$

Note → $i=\sqrt{-1}$, $i^2 = (\sqrt{-1})^2 \Rightarrow -1$

$i^3 = i^2 \times i \Rightarrow -i$

$i^4 = i^2 \times i^2 \Rightarrow (-1)\times(-1) \Rightarrow 1$

Example → If $z_1 = 5+7i$ and $z_2 = 3+4i$, find z_1+z_2, z_1-z_2, z_1z_2 and $\frac{z_1}{z_2}$

Solution → $(z_1+z_2) = 5+7i+3+4i$

$\Rightarrow (5+3)+i(7+4)$

$\Rightarrow 8+11i$

$(z_1-z_2) \Rightarrow (5+7i)-(3+4i)$

$\Rightarrow 5+7i-3-4i$

$\Rightarrow (5-3)+(7i-4i)$

$\Rightarrow 2+3i$

$z_1z_2 \Rightarrow (5+7i)(3+4i)$

$\Rightarrow 5(3+4i)+7i(3+4i)$

$\Rightarrow 15+20i+21i+28i^2$

$\Rightarrow (15-28)+(20i+21i)$

$\Rightarrow -13+41i$

$\frac{z_1}{z_2} \Rightarrow \frac{5+7i}{3+4i}$

$\Rightarrow \frac{5+7i}{3+4i} \times \frac{3-4i}{3-4i}$

$\Rightarrow \frac{5(3-4i)+7i(3-4i)}{(3)^2-(4i)^2}$

$\Rightarrow \frac{15-20i+21i-28i^2}{9+16}$

$\Rightarrow \frac{15+28+i}{25} \Rightarrow \frac{43+i}{25}$

✱ Conjugate of a complex number –
If $z = x+iy$ is any complex number, then the complex number $x-iy$ is called the conjugate of the complex number z and is written as $\bar{z}$.

Example → Conjugate of $2+3i$.
Solution → $2-3i$ is the conjugate

✱ Modulus of a complex number.
• The modulus of a complex number $z = x+iy$ is the non-negative real number $\sqrt{x^2+y^2}$ and is defined by $|z|$.
Thus, $|z| = +\sqrt{x^2+y^2}$

Example → $|2+3i| \Rightarrow +\sqrt{2^2+3^2} = +\sqrt{13} = |z|$

✱ Additive and Multiplicative inverse of a complex number –

- If z_1 & z_2 be two complex numbers, and if $z_1 + z_2 = 0$, then each is said to be the additive inverse of the other. Thus, additive inverse of a complex number z is equal to $-z$.

Example → find the additive inverse of $2+3i$.

Ans → $z = 2+3i$

$-z = -(2+3i) = -2-3i$

→ Multiplicative inverse → $z = 1/z$

Example → find the Multiplicative inverse of $2+3i$.

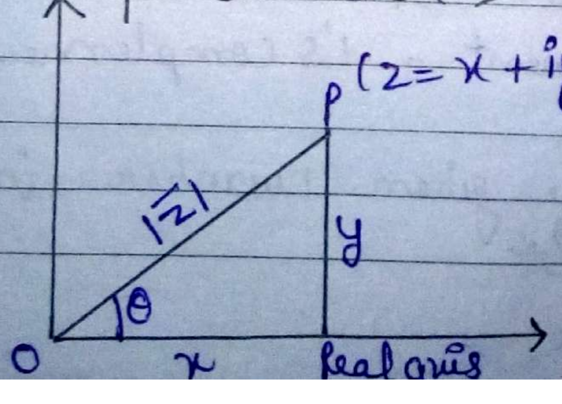
Ans → Multiplicative inverse of $z = \frac{1}{z}$

$$z = \frac{1}{2+3i} \times \frac{2-3i}{2-3i} \Rightarrow \frac{2-3i}{(2)^2-(3i)^2}$$

$$z \Rightarrow \frac{2-3i}{4+9} \Rightarrow \frac{2-3i}{14}$$

✱ The Order Property –

Graphical representation →



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* Polar form of a complex number –
- Every non-zero complex number $x+iy$ can be put in the form $r(\cos\theta + i\sin\theta)$, where r and θ are both real numbers.
- This is called the polar form of the complex number z.
- Let, $x+iy = r(\cos\theta + i\sin\theta)$
then, $r = +\sqrt{x^2+y^2}$ is called the modulus z, and $\theta = \tan^{-1}\frac{y}{x}$ is called the amplitude or the argument of z.

Ex → $z = 2+0i$ write its polar form.

$$x+iy = r(\cos\theta + i\sin\theta)$$

$$r = \sqrt{x^2+y^2}$$

$$r = \sqrt{(2)^2+(0)^2} \Rightarrow \sqrt{4+0} = \sqrt{4} = 2$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{0}{2} = \tan^{-1}0$$

$$\theta = 0$$

$$2+0i = 2(\cos 0 + i\sin 0)$$

$$= 2(1 + i\times 0)$$

$$= 2$$

* 1's complement →
It is define as $1 \to 0$ and $0 \to 1$

* 2's complement →
2's complement = 1's complement + 1

Example → Convert the given number in 1's and 2's complement?

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(i) 1011000

1's complement → 0100111

2's complement → ~~0~~ 01000111

$$\begin{array}{r} 01000111 \\ +\ 1 \\ \hline 01001000 \end{array}$$

(ii) 1001001

1's complement → 0110110

2's complement → 0110110

$$\begin{array}{r} 0110110 \\ +\ 1 \\ \hline 0110111 \end{array}$$

(iii) 01100

1's complement → 10011

2's complement → 10011

$$\begin{array}{r} 10011 \\ +\ 1 \\ \hline 10100 \end{array}$$

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