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PERMUTATION AND COMBINATION

→ $\lfloor 5$ or $5!$

$\lfloor 5 = 5 \times 4 \times 3 \times 2 \times 1 = 120$

represented by $= {}^nP_r = \frac{\lfloor n}{(n-r)!}$

Factorial → Product of "n" element (without repeated). $\lfloor n$ or $n! = 1 \times 2 \times 3 \times 4 \ldots n$
factorial is the product of 'n' element

Permutation → Permutation is a arrangement of 'r' element of set 'n' element that permutation is called of 'r' permutation of $r = n$, then it is called permutation it is denoted by ⇒ $P(n, r)$ or ${}^nP_r = \frac{\lfloor n}{\lfloor n-r}$

Case first → The number distinct permutation that can be from of the collection of 'n' elements in which first element appear in K_1 times and second element appear in K_2 times and so on is,

n = Total no. of element
K = element
r = Repeated element

${}^nP_r = \frac{n!}{K_1!\, K_2! \ldots K_r!}$ (repetation)

Example → MATHEMATICS

M = 2 | T = 2
A = 2 |

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$^{11}P_r \Rightarrow \frac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2} \Rightarrow$.

IInd Case → Circular permutations ⟶

The permutation of element circle is called Circular permutation. The number of circular permutations can be find as —

$$^nP_r = (n-1)!$$

Ques → Prove that $^nP_{n-1} = {}^nP_n$

L.H.S → $\frac{\lfloor n}{\lfloor n-(n-1)} \Rightarrow \frac{\lfloor n}{\lfloor n-n+1}$

$= \lfloor n$

R.H.S ⇒ ^nP_n

$\rightarrow \frac{\lfloor n}{\lfloor n-n} \Rightarrow \frac{\lfloor n}{\lfloor 0} \Rightarrow \lfloor n$

So, R.H.S = L.H.S.

Ques - Find the value of 'n'?

$$^7P_n = 2 \times {}^7P_{n-2}$$

As we know that,

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\frac{7!}{(7-n)!} = 2 \times \frac{7!}{[7-(n-2)]!}$$

$$\frac{\cancel{7!}}{(7-n)!} = \frac{2 \times \cancel{7!}}{(9-n)!}$$

$$\Rightarrow \frac{1}{\cancel{(7-n)!}} = \frac{2}{(9-n)(8-n)\cancel{(7-n)!}}$$

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$(9-n)(8-n) = 2$

$72 - 9n - 8n + n^2 = 2$

$n^2 - 17n + 70 = 0$

$n^2 - 10n - 7n + 70$

$n(n-10) - 7(n-10) = 0$

$(n-10)(n-7) = 0$

$n = 10$ and $n = 7$

~~Ques~~ - find the Value of ${}^nP_1 + {}^nP_2$

${}^nP_r = \frac{\lfloor n}{\lfloor n-r}$

So, $\frac{\lfloor n}{\lfloor n-1} + \frac{\lfloor n}{\lfloor n-2}$

$\frac{\lfloor n}{\lfloor n-1} + \frac{\lfloor n}{(n-2)\lfloor n-1}$

$\frac{\lfloor n(n-2) + \lfloor n}{(n-2)\lfloor n-1}$

$\Rightarrow$

${}^nP_r \rightarrow \frac{n!}{(n-r)!}$

Ques → find the Value of ${}^nP_1 + {}^nP_2$

$\frac{\lfloor n}{\lfloor n-1} + \frac{\lfloor n}{\lfloor n-2}$

$\Rightarrow \frac{\lfloor n}{(n-1)\lfloor n-2} + \frac{\lfloor n}{\lfloor n-2}$

$\cancel{\frac{\lfloor n + \lfloor n(n-1)}{(n-1)\lfloor n-2}}$

$\Rightarrow \frac{\lfloor n}{\lfloor n-2}\left[\frac{1}{n-1} + \frac{1}{1}\right]$

$= \frac{\lfloor n}{\lfloor n-2}\left[\frac{1+n-1}{(n-1)}\right]$

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$$\frac{n!}{(n-2)!} \times \frac{n}{n-1} \Rightarrow \frac{n\,\cancel{(n-1)}\,\cancel{(n-2)!}}{\cancel{(n-2)!}} \times \frac{n}{\cancel{(n-1)}}$$

$$= n^2 \text{ Ans}$$

Ques → find the value of the following –

1. ^nP_n

→ $\frac{\lfloor n}{\lfloor n-n} \Rightarrow \frac{\lfloor n}{\lfloor 0} \Rightarrow \lfloor n$

2. $^nP_{(n-1)}$

$\Rightarrow \frac{n!}{(n-n+1)!} \Rightarrow n!$ Ans

3. $n \cdot {}^{n}P_{(n-1,\ n-1)}$

→ $\frac{n!}{(n-1-n+1)!} \Rightarrow n! \quad n \cdot \frac{(n-1)!}{(n-1-(n-1))!}$

$= n \cdot (n-1)!$

Ques → Find in how many ways can 6 boys sited at roundtable.

→ No. of ways → $(n-1)!$

→ $(6-1)!$

→ $5!$

→ $5 \times 4 \times 3 \times 2 \times 1$

⇒ 120

Ques. find the distinct permutation from the word 'RADAR'.

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$\rightarrow$ $^nP_r = \frac{n!}{\quad}$

$n = 5$

$A = 2$

$R = 2$

$^5P_r = \frac{\angle 5}{\angle 2 \angle 2} \Rightarrow \frac{5 \times \cancel{4}^{2} \times 3 \times \cancel{2} \times 1}{\cancel{2} \times 1 \times \cancel{2} \times 1}$

$\Rightarrow 30$

Ques $\rightarrow$ In how many boys can 5 boys and 4 Girls sit around a table so that no two girls can sit together.

Boys $= \angle n-1$

$= \angle 5-1$

$= 4 \times 3 \times 2 \times 1 = 24$

Girls $\Rightarrow \angle n-1$

$\Rightarrow \angle 4-1 \Rightarrow \angle 3$

$\Rightarrow 3 \times 2 \times 1 \Rightarrow 6$

Q. In how many 5 boys and 5 girls can be sited in row so that (i) two girls are sit together.

(ii) All girls sit together and all boys sit together.

$\rightarrow$ ① $5! \cdot 5! \Rightarrow 14400$ Ans.

② $5! \cdot 5! + 5! \cdot 5! = 14400 + 14400$

$= 28800$

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Ques→ How many different words can be form in with the letter on words—

MISSISSIPPI

$$^nP_r = \frac{\lfloor n}{\lfloor K_1 \lfloor K_2 \dots \lfloor K_r}$$

$n = 11$ | $S = 4$

$I = 4$ | $P = 2$

$$^{11}P_r = \frac{\lfloor 11}{\lfloor 4 \lfloor 4 \lfloor 2} \Rightarrow \frac{11 \times 12 \times 10 \times 8 \times 8 \times 7 \times 6 \times 5 \times 4 \times 4}{4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1}$$

$\Rightarrow 3 \times 2 \times 1 \times 11 \times 3 \times 10 \times 7 \times 3 \times 5 \times 2$

$= \cancel{34650}$ 405860

Ques→ In how manys can 6 boys stand in the row.

$\lfloor 6$ in row

Ans→ $6 \times 5 \times 4 \times 3 \times 2 \times 1 \Rightarrow 720$

Ques→ find the number of different 4 letter words that can be formed from the word WONDER.

$n = 6$

$$^nP_r = \frac{n!}{r!}$$

$$^6P_4 = \frac{6!}{4!} \Rightarrow \frac{6 \times 5 \times \cancel{4!}}{\cancel{4!}} \Rightarrow 30$$

Ques→ In how many different words can be formed from the word TRIANGELS

⇒

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Ques → In how many ways 4 mathematics book, 3 Computer Science book and 3 economics book we arrange in a Sale so that all book of Some subject remain together.

→ $M = 4 \rightarrow \lfloor 4$

$C.S = 3 \rightarrow \lfloor 3$

$Eco = 3 \rightarrow \lfloor 3$

MEC, EMC, ECM, CME, CEM, MCE

So. No of permutation $\Rightarrow 6 \times \lfloor 4 \times \lfloor 3 \times \lfloor 3$

$\Rightarrow 6 \times 4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1$

$\Rightarrow \cancel{6 \times 6 \times 4 \times 9}$

$\Rightarrow \cancel{36 \times 36}$ 5186

Ques. There are 4 mathematics book 4 physics book and 3 computer book in how many ways they can be arranged on a cell so that book of Same Subject are not seperated?

Solution. $M = 4$, $P = 4$ and $C = 3$

MPC, MCP, PMC, PCM, CMP, CPM

No. of permutations $\rightarrow 6 \times \lfloor 4 \times \lfloor 4 \times \lfloor 3$

$\rightarrow 6 \times 4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1$

$\Rightarrow 6 \times 24 \times 24 \times 6$

$\Rightarrow 20736$ Ans

✱ COMBINATION —

Combination → It is a subset of 'r' elements from the set of 'n' elements.

Ex → Bunch of flowers.

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It is represented by $\rightarrow {}^nC_r$

$C(n,r)$.

No. of combination $= {}^nC_r$ or $C(n,r)$

$$\Rightarrow \frac{\underline{|n}}{\underline{|r}\,\underline{|(n-r)}}$$

when $0 \le r \le n$

If $r = n$ then combination

Ex $\rightarrow$ Prove that $C(n,r) = C(n, n-r)$ when $0 \le r \le n$

$\rightarrow$ ${}^nC_r = {}^nC_{n-r}$

$$\frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} = \frac{\underline{|n}}{\underline{|n-r}\,\underline{|(n-n+r)}}$$

$$\frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} = \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}$$

RHS = RHS

Ques : Prove that $C(n,r) = C(n-1, r-1) + C(n-1, r)$

Where $0 \le r \le n$

$\longrightarrow$ L.H.S $\Rightarrow C(n-1, r-1) + C(n-1, r)$

$$\frac{(n-1)!}{(r-1)!\,[\{n-1-(r-1)\}]!} + \frac{(n-1)!}{r!\,(n-1-r)!}$$

$$\Rightarrow \frac{(n-1)!}{(r-1)!\,[n-1-r+1)!} + \frac{(n-1)!}{r!\,(n-1-r)!}$$

$$\Rightarrow \frac{(n-1)!}{(r-1)!\,(n-r)!} + \frac{(n-1)!}{r!\,(n-1-r)!}$$

$$\Rightarrow \frac{(n-1)!}{(r-1)!\,(n-r)(n-r-1)!} + \frac{(n-1)!}{r(r-1)!\,(n-1-r)!}$$

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$$\frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\frac{1}{n-r}+\frac{1}{r}\right]$$

$$\frac{(n-1)!}{r(r-1)!\,(n-r-1)!}\left[\frac{\cancel{r}+n-\cancel{r}}{(r)(n-r)}\right]$$

$$\frac{(n-1)!}{\cancel{r}(r-1)!\,(n-r-1)!}\left[\frac{n}{r(n-r)}\right]$$

$$\frac{n(n-1)!}{r(r-1)!\,(n-r)(n-r-1)!}$$

$$\Rightarrow \frac{n!}{r!\,(n-r)!}$$

$\Rightarrow C(n,r)$

LHS = RHS

Ques → Find the Value of $C(8,5)$.

An. $^8C_5 \Rightarrow \frac{8!}{5!(8-5)!} \Rightarrow \frac{8\times7\times6\times\cancel{5!}}{\cancel{5!}\times3!} \Rightarrow \frac{8\times7\times\cancel{6}}{\cancel{3}\times\cancel{2}\times1} \Rightarrow 56$

Ques → If $C(n,5)=C(n,3)$ then find the value of n.

$$^nC_5 = {}^nC_3$$

$$\frac{\cancel{n!}}{5!(n-5)!} = \frac{\cancel{n!}}{3!(n-3)!}$$

$$\frac{1}{5\times4\times\cancel{3!}\times(n-5)!} = \frac{1}{\cancel{3!}\times(n-3)!}$$

$$\frac{1}{20(n-5)!} = \frac{1}{(n-3)!}$$

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$$\frac{1}{20(n-\cancel{5})!} = \frac{1}{(n-3)(n-4)\cancel{(n-5)}!}$$

$(n-3)(n-4) = 20$

$n^2 - 4n - 3n + 12 = 20$

$n^2 - 7n - 8 = 0$

$n^2 - 8n + n - 8 = 0$

$n(n-8) + 1(n-8) = 0$

$(n-8)(n+1) = 0$

So, $n = 8$ but $n \neq -1$

Ques- In how many ways can a Comitte of 5 be formed from 8 people.

A. $^8C_5 \Rightarrow \frac{8!}{5!(8-5)!} = 56$

Ques- In how many ways a Comitte of 6 members can be constituted from 8 men and 5 women constituting 3 men and 3 women.

men $\Rightarrow$ $^8C_3$

Women $\Rightarrow$ $^5C_3$

$\Rightarrow$ $^8C_3 \times {}^5C_3$

$\Rightarrow \frac{8!}{3!(8-3)!} \times \frac{5!}{3!(5-3)!}$

$\Rightarrow \frac{8 \times 7 \times \overset{2}{\cancel{6}} \times \cancel{5!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{5!}} \times \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3!}}{\cancel{3!} \times \cancel{2} \times 1}$

$\Rightarrow 56 \times 10$

$= 560$ A.

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Case I → The number of combinations of 'n' elements 'r' at a time. In which 'm' particular elements are —

(1) Always included is $C(n-m, r-m)$

(2) Never included $C(n-m, r)$ $m \le n$

Ques → In how manys can a committe of 5 persons from out of 9 persons. If two particulars persons always selected to the committe.

Ans → $^7C_3 = \frac{\lfloor 7}{\lfloor 3 \, \lfloor 7-3} \Rightarrow \frac{7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{4!}} \Rightarrow 35$

because 2 already selected

Ques → There are 12 persons in a party. And each 2 of them hands with each other. How many handshakes in the party.

Ans → $^{12}C_2 = \frac{12!}{2!\,(12-2)!} \Rightarrow \frac{12 \times 11 \times \cancel{10!}}{2! \times \cancel{10!}} \Rightarrow \frac{\cancel{12}^{6} \times 11}{\cancel{2} \times 1} = 66$ Ans

Ques → In how many ways a football 11 can be selected from among 17 players. If

(i) 5 players are always include in the 11.

(ii) Two particular players always ~~are~~ excluded

Ans → (i) $^{12}C_6 \Rightarrow \frac{12!}{6!\,(12-6)!} \Rightarrow \frac{\cancel{12}^{2} \times 11 \times \cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8}^{2} \times 7 \times \cancel{6!}}{\cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}}$

$\Rightarrow 11 \times 2 \times 3 \times 2 \times 7$

$\Rightarrow 924$

(ii) $^{15}C_{11} \Rightarrow \frac{15!}{11!\,(15-11)!} \Rightarrow \frac{\cancel{15}^{5} \times \cancel{14}^{7} \times 13 \times \cancel{12}^{3} \times \cancel{11!}}{\cancel{11!} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

$\Rightarrow 5 \times 7 \times 13 \times 3 = 1365$

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Exaple → A Box contains 8 red balls, 5 blue balls. In how many can 4 balls withdrawn from the box—

(1) They are of any colour
(2) Two balls are red and two blue
(3) All the balls are of same colour

Ans. $^{13}C_4 \Rightarrow \dfrac{13!}{4!\,(13-4)!} = \dfrac{13 \times \cancel{12}^{3} \times 11 \times \cancel{10}^{5} \times \cancel{9!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{9!}}$

$\Rightarrow 13 \times 11 \times 5$

$\Rightarrow 715$

(ii) $^{8}C_2 \times {}^{5}C_2 \Rightarrow \dfrac{8!}{2!\,(8-2)!} \times \dfrac{5!}{2!\,(5-2)!}$

$\Rightarrow \dfrac{\cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{2} \times 1 \times \cancel{6!}} \times \dfrac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{2} \times 1 \times \cancel{3!}}$

$\Rightarrow 28 \times 10$

$\Rightarrow 280$

(iii) $^{8}C_4 + {}^{5}C_4 \Rightarrow \dfrac{8!}{4!\,(8-4)!} + \dfrac{5!}{4!\,(5-4)!}$

$\Rightarrow \dfrac{\cancel{8}^{2} \times 7 \times \cancel{6}^{2} \times 5 \times \cancel{4!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{4!}} + \dfrac{5 \times \cancel{4!}}{\cancel{4!} \times 1}$

$\Rightarrow 70 + 5$

$\Rightarrow \cancel{38}\ 75$ Ans.

Ques → In how many ways can a committe be constitute of 3 proffessors and 2 reader.

from a group of 5 professors and 8 reader.

Ans- $^5C_3 \times {}^8C_2 \Rightarrow \frac{5\times4\times\cancel{3!}}{\cancel{3!}\times(5-3)!} \times \frac{8!}{2!(8-2)!}$

$\Rightarrow \frac{5\times\cancel{4}^2}{\cancel{2}\times1} \times \frac{\cancel{8}^4\times7\times\cancel{6!}}{\cancel{2}\times1\times\cancel{6!}}$

$\Rightarrow 10\times28 = 280$

Example- In how many ways committe consisting of 4 persons including a giving chairman can be formed from a group of 10 persons.

Ans. $^9C_3 \Rightarrow \frac{9!}{3!(9-3)!} \Rightarrow \frac{\cancel{9}^3\times\cancel{8}^4\times7\times\cancel{6!}}{\cancel{3}\times\cancel{2}\times1\times\cancel{6!}} \Rightarrow 28\times3$

$\Rightarrow 84$ Ans.

Case II → The total number of combination of $n_1+n_2+n_3$ up to elements pair n_1 are alike, n_2 are alike and so on, taken any number at a time in ~~n_1+n~~. $(n_1+1)(n_2+1)(n_3+1)\ldots$

Example- In how many ways can a cricket team of 11 players can be selected from 25 players in which there are 10 batsmen, 8 ballers, 5 all rounders and 2 wicket keepers assume that the team 11 has 5 batsmen, 3 all rounders, 2 ballers and 1 wicket keeper

Soln- $B = 10 \longrightarrow {}^{10}C_5$

Ballers → 8 → $^8C_2$

5 all rounder → $^5C_3$

2 wicket keeper → $^2C_1$

$^{10}C_5 \times {}^8C_2 \times {}^5C_3 \times {}^2C_1$

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$$\frac{10!}{5!(10-5)!} \times \frac{8!}{2!(8-2)!} \times \frac{5!}{3!(5-3)!} \times \frac{2!}{1!(2-1)!}$$

$$\Rightarrow \frac{10!}{5! \times 5!} \times \frac{8!}{2! \times 6!} \times \frac{5!}{3! \times 2!} \times \frac{2!}{1! \times 1!}$$

$$\Rightarrow \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5! \times 5 \times 4 \times 3 \times 2 \times 1} \times \frac{8 \times 7 \times 6!}{2 \times 1 \times 6!} \times \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times 2$$

$$\Rightarrow \frac{3 \times 7 \times 3 \times 8}{2} \times 28 \times 10 \times 2$$

$$\Rightarrow 63 \times 28 \times 80 \Rightarrow 141120$$

Ques In how many ways can be selection out of 12 mathematics books, 7 eco book & 10 chemistry books, we made if at least one of these books be selected?

Maths = (12+1)
Economics = (7+1)
Chemistry = (10+1)
$\Rightarrow (12+1)(7+1)(10+1) - 1$
$\Rightarrow 13 \times 8 \times 11 - 1$
$\Rightarrow 13 \times 88 - 1$
$\Rightarrow 1144 - 1$
$\Rightarrow 1143$

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✱ Pigeon Hole Principle —

- If n items are put into m Containers with ~~xx~~ n > m then at least one Container must contain more then one item.

Ques → A patient is given a description of 45 tablets with the instructions to take at least one tablet per day for 30 days prove that there must be a period of consecutive days during which the patient takes a total of exactly 14 tablets.

Sol^n → $1 \leq a_1 \quad a_2 \quad a_3 \ldots\ldots a_{30} \leq 4$

$1 \leq (a_1 + 14) < (a_2 + 14) < (a_3 + 14) \ldots < (a_{30} + 14) \leq 45 + 14$

Total No. of tablets = 59 tablets

Total No. of days = 60 days

n

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