PRINCIPLE OF MATHEMATICAL INDUCTION

Ques → $P(n) = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$.

Solⁿ → $P(n) = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

$P(1) = 1 + 2 + 3 + \dots + 1 = \frac{1(1+1)}{2}$

L·H·S = 1

$R·H·S = \frac{1(1+1)}{2} = 1$

So, L·H·S = R·H·S for n = 1

It is true for n = 1

Let, It is true for n = K.

$P(K) = 1 + 2 + 3 + \dots + K = \frac{K(K+1)}{2}$ —(1)

Suppose, it is also true for n = K+1

$P(K+1) = 1 + 2 + 3 + \dots + K + K + 1 = \frac{(K+1)(K+2)}{2}$

Then from eqⁿ(1) —

$\rightarrow \cancel{P(K+1) = 1 + 2 +} \frac{K(K+1)}{2} + K + 1 = \frac{(K+1)(K+2)}{2}$

$\rightarrow \frac{K(K+1) + 2(K+1)}{2} = \frac{(K+1)(K+2)}{2}$

$\rightarrow \frac{(K+2)(K+1)}{2} = \frac{(K+1)(K+2)}{2}$

So, L·H·S = R·H·S.

Ques. Show by principle mathematical induction that the sum of first odd natural number is n^2.

solution → $P(n) = 1 + 3 + 5 + \dots + 2n - 1 = n^2$

$P(1) = 1 + 3 + 5 + \dots + 2(1) - 1 = (1)^2$

$P(1) = 1 + 3 + 5 + \dots + 1 = 1$

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L.H.S = R.H.S

It is true for n=1.

Let, It is true for n=k.

$P(K) = 1+3+5+ \dots + 2K-1 = K^2$ ——— (1)

Suppose, it is also true for n=K+1

$P(K+1) = 1+3+5+ \dots 2K-1 + 2K+2-1 = (K+1)^2$

$P(K+1) = 1+3+5+ \dots 2K-1 + 2K+1 = (K+1)^2$

from eqn(1)

$(K^2+2K+1) = (K+1)^2$

$(K+1)^2 = (K+1)^2$

So, it is also true for K+1.

Ques Prove by Mathematical Induction that —

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

Solution → $P(n) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

$P(1) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \dots + \frac{1}{1(1+1)} = \frac{1}{1+1}$

$P(1) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \dots + \frac{1}{2} = \frac{1}{2}$

$P(K) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \dots + \frac{1}{K(K+1)} = \frac{K}{K+1}$ —(1)

$P(K+1) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)} = \frac{K+1}{K+2}$

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PAGE NO.

Then, from eqn (1)

$$\rightarrow \ldots\ldots \frac{k}{1(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

$$\rightarrow \ldots\ldots \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

$$\ldots\ldots \frac{k^2+2k+1}{}$$

$$\rightarrow \ldots\ldots \frac{1}{k+1}\left(k + \frac{1}{k+2}\right) = \frac{k+1}{k+2}$$

$$\rightarrow \ldots\ldots \frac{1}{k+1}\left[\frac{k(k+2)+1}{k+2}\right] = \frac{k+1}{k+2}$$

$$\rightarrow \ldots\ldots \frac{1}{k+1}\left[\frac{k^2+2k+1}{k+2}\right] = \frac{k+1}{k+2}$$

$$\rightarrow \ldots\ldots \frac{(k+1)^{\cancel{2}}}{\cancel{(k+1)}(k+2)} = \frac{k+1}{k+2}$$

$$\rightarrow \ldots\ldots \frac{k+1}{k+2} = \frac{k+1}{k+2}$$

So, LHS = RHS

Ques. Prove by Mathematical Induction
$10^n + 3\cdot 4^{n+2} + 5$ is divisible by 9

Soln → $10^n + 3\cdot 4^{n+2} + 5 = 9a$

put, $n=1$

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$10 + 3\cdot 4^3 + 5 \Rightarrow 9 \times 23 = 207$

it is divisible by 9.

PAGE NO. :

~~P(n) is divisible by~~ P(n) is true for n = 1

Let is it true for n = K

$P(K) = 10^K + 3.4^{K+2} + 5 = 9q$

$\Rightarrow 3.4^{K+2} = 9q - 10^K - 5$

now we have prove n = K + 1

$P(K+1) = 10^{K+1} + 3.4^{(K+1+2)} + 5 = 9q$

$10^{K+1} + 3.4^{(K+3)} + 5 = 9q$

$10^{K+1} + 3.4^{(K+2)}.4 + 5 = 9q$

$\Rightarrow 10^K.10 + 4(9q - 10^K - 5) + 5 = 9q$

$\Rightarrow 10^K.10 + 36q - 4.10^K - 20 + 5 = 9q$

$\Rightarrow 10^K(10-4) + 36q - 15 = 9q$

$\Rightarrow 10^K.6 + 36q - 15 = 9q$

$\Rightarrow 9[\frac{2}{3}.10^K + 4q - \frac{5}{3}] = 9q$

L.H.S = R.H.S

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