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Relations & Functions

* Relations → Subset of order pair of $A \times B$

$A = \{a, b, c\}$
$B = \{1, 2, 3\}$
$A \to B$

$$A \times B = \begin{cases} (a,1), & (a,2), & (a,3) \\ (b,1), & (b,2), & (b,3) \\ (c,1), & (c,2), & (c,3) \end{cases}$$

• Pair of elements of set A to B is called order pair of $A \to B$.

* Cartesian Product →
• Set of all order pair of $A \to B$.

* No. of relations — 2^{mn}
• m = no. of elements in set A, $n(A)$
• $n = n(B)$ = no. of elements in set B.

Ques- If A & B having n elements of the set then no. of relations from $A \to B$ is.

(a) 2^n
(b) 2^{n^2} ✓
(c) 2^{n+1}
(d) n^2

→ $2^{mn} \to 2^{n \times n} \to 2^{n^2}$

Ques. If R is a relation in which we have r no. of relation how many non-empty subsets it have —

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(i) 2^r (ii) 2^{r-1}

(iii) $2^r - 1$ (iv) 2^{r+1}

Domain & Range –

$R = \{(a,1), (b,2), (c,3)\}$

Domain → $\{a, b, c\}$ → Set of all first element of relation

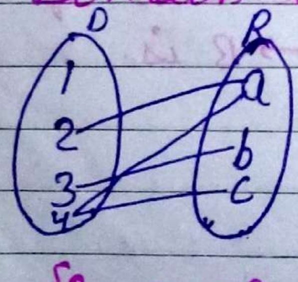
Range → $\{1, 2, 3\}$ → Set of all Second elements

Ques. If $A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$
which of the following relations form $A \to B$
Give Reason –

(a) $R_1 \to \{(1,4), (1,5), (1,6)\}$
(b) $R_2 \to \{(1,5), (3,6), (2,4)\}$
(c) $R_3 \to \{(3,4), (2,6), (3,6), (1,4), (1,5)\}$
(d) ~~$R_4 \to$~~ $\{(5,1), (2,6), (2,4), (4,2)\}$

Ques $A = \{1, 2, 3, 4\}$
$B = \{a, b, c\}$
$R = \{(2,a), (3,b), (4,a), (4,c)\}$
find the Domain and Range

→



Domain – $\{2, 3, 4\}$
Range – $\{a, b, c\}$

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✱ Inverse Relation →

- It is a subset of $B \times A$.

  $A = \{1, 2, 3\}$

  $B = \{a, b, c\}$

  $B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3)\}$

✱ Types of Relation —

(i) Identity relation —

A relation is called identity relation of a set 'A' when each element of A is related to itself only. for e.g. → $A = \{a, b, c\}$

$R = \{(a,a), (b,b), (c,c)\}$

(ii) Reflexive Relation →

Reflexive relation a relation on set A is said to be reflexive if every element of A is related to itself.

$R = \{(a,a), (b,b), (c,c), (a,b), (b,c), (c,a)\}$

Ques - Let, $A = \{1, 2, 3\}$ then examine $R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (3,2)\}$ is reflexive set.

A — Yes, it is a reflexive

(iii) Symmetric Relation —

- A relation R on a set A is said to be symmetric if and only — if $(a,b) \in R$ then $(b,a) \in R$.

  $A = \{a, b, c\}$

  $R = \{(a,b), (a,a), (b,a)\}$

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4) Transitive Relation –

- A relation R on a set A is said to be transitive if and only if → if $(x, y) \in R$ and $(y, z) \in z$ then $(x, z) \in R$,

Equivalence Relation →

A relation on the set A is called equivalence if R is reflexive, symmetric and transitive.

Example → Let R_1 be A relation on a set of all lines in a plane defined by $(l_1, l_2) \in R \leftrightarrow R_1$ is parallel to l_2. Show that R is an equivalence relation

Ans → Reflexive → Let $l \in L$

$(l_1, l_1) \in R \rightarrow l_1$ is parallel to l_1.

Every line of L is parallel to itself. R is reflexive relation.

Symmetric → Let, $l_1, l_2 \in L$

$(l_1, l_2) \in R \rightarrow l_1$ is parallel to l_2

l_2 is parallel to l_1.

R is Symmetric relation.

Transitive → Let l_1, l_2 and $l_3 \in L$ such that

$(l_1, l_2) \in R \rightarrow l_1$ is parallel to l_2

$(l_2, l_3) \in R \rightarrow l_2$ is parallel to l_3

So, $(l_1, l_3) \in R$

R is a transitive relation

Since, R is a reflexive, transitive and symmetric relation so it is a equivalence relation.

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Ques → Show that the relation is congruent to the set of all triangles in an equivalence relation.

Sol^n → Reflexive → Let $\Delta \in A$ such that,

$(\Delta, \Delta) \in R \rightarrow \Delta$ is congruent to Δ

every Δ is congruent to itself

So, R is reflexive for all $\Delta \in A$.

Symmetric → Let $\Delta_1, \Delta_2 \in A$ such that

$(\Delta_1, \Delta_2) \in R \rightarrow \Delta_1$ is congruent to Δ_2

Δ_2 is congruent to Δ_1

R is ~~reflexive~~ $\forall \Delta_1, \Delta_2 \in A$.

Transitive → Let Δ_1, Δ_2 & $\Delta_3 \in A$ such that

$(\Delta_1, \Delta_2) \in R \rightarrow \Delta_1$ is congruent to Δ_2

$(\Delta_2, \Delta_3) \in R \rightarrow \Delta_2$ is congruent to Δ_3

So, $(\Delta_1, \Delta_3) \in R$

R is a transitive relation.

So, it is equivalence relation because it is transitive, symmetric & reflexive.

Ques → Show that the relation R on the set Z defined by $(x, y) \in R \rightarrow (x-y)$ is divisible by n.

Solution Reflexive → Let $x \in Z$ such that

$(x, x) \in R \rightarrow x - x$ is divisible by n bec. $x - x$ is 0. So R is reflexive $\forall x \in I$

Symmetric → Let $x, y \in Z$ such that

$(x, y) \in R \rightarrow (x-y)$ is divisible by n.

$\rightarrow (x-y)$ is divisible by n.

$\rightarrow (y-x)$ is divisible by n

$(y, x) \in R$, so R is symmetric $\forall (x, y) \in I$

Transitive → Let $x, y, z \in Z$ such that $(x, y) \in R$ and $(y, z) \in R$.

$(x,y) \in R \rightarrow (x-y)$ is divisible by n, $x-y = np$ —(i)
$(y,z) \in R \rightarrow (y-z)$ is divisible by n, $y-z = nQ$ —(ii)
from (i) & (ii)

$$x - np - z = nQ$$
$$x - z = nQ + np$$
$$x - z = n(P+Q)$$

$x - y = np$
$-z + y = nQ$
$x - z = n(P+Q)$

Let $p + Q = r$
so, $x - z = nr$
$(x-z)$ is divisible by n
$(x,z) \in R$. It is transitive $\forall\ x, y, z \in I$.
So. it is a equivalence relation because they are reflexive, Symmetric and transitive

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✱ Anti-Symmetric Relation —

Solu

- A relation 'R' on the set 'A' is said to be Anti-Symmetric relation if and only if $(a,b) \in R$ and $(b,a) \in R \leftrightarrow a = b\ \forall\ a, b \in A$

✱ Partial Order Relation —

If a relation reflexive, Anti-symmetric & Transitive then that relation is called partial order Relation.

Ques - The relation "$\geq$" on the set of integers then prove that R is partial order relation?

Sol^n → Reflexive → Let $a \in z$ such that
$(a,a) \in R \leftrightarrow a$ is equal to $a \leftrightarrow$
R is reflexive relation.
Anti-Symmetric → $(a,b) \in z$ such that
$(a,b) \in R$ and $(b,a) \in R$
$a \geq b$ & $b \geq a \leftrightarrow a = b$

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—(i) So, R is Anti-Symmetric relation $\forall (a,b) \in Z$.

—(ii) Transitive → Let $a, b, c \in Z$ such $(a,b) \in R$ and $(b,c) \in R$.

$(a,b) \in R \rightarrow a \geq b$ —— (1)

$(b,c) \in R \rightarrow b \geq c$ —— (2)

So, $a \geq c$. from (1) & (2)

So, R is transitive relation $\forall a, b, c \in Z$.

Therefore R is reflexive, symmetric & anti-symmetric. So, it is partial order relation.

Ques → Prove that the relation R on the set $N \times N$ defined by $(a,b) R (c,d) \longleftrightarrow a+d = b+c$ $\forall (a,b), (c,d) \in N \times N$ is an equivalence relation

Solution → Reflexive → Let $(a,b) \in N \times N$ such that

$(a,b) R (a,b) \longleftrightarrow a+b = b+a$

R is reflexive $\forall (a,b) \in N \times N$.

Symmetric → Let $(a,b) \in N \times N$ and $(c,d) \in N \times N$.

$(a,b) R (c,d) \longleftrightarrow a+d = b+c$

$(b,a) R (c,d) \longleftrightarrow d+a = c+b$

$a+b = b+c$

$c+b = d+a$

$(c,d) R (a,b)$

Congruence Modulo m —

Two integers a and b are said to be congruent modulo if $a-b$ is divisible by m.

Ex → $17 \equiv 2 \pmod 5$

Digraph of Relation →

- Let A be the finite set and R be a relation on the set A.

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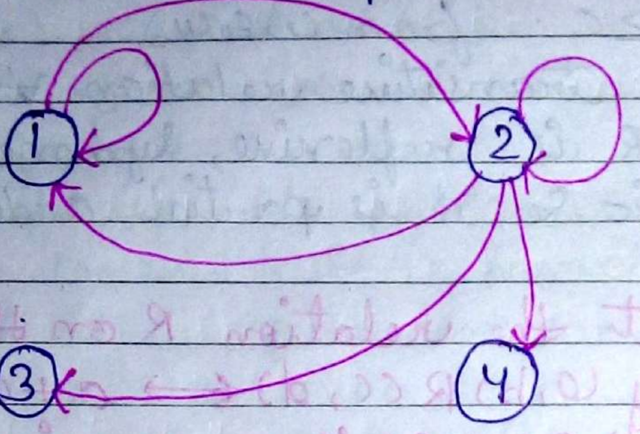
Then the relation R Can be represented by the pictures that graph is called digraph of the relation. In this no. of elements are represented in small circle and relation of the elements represented by arrow lines.



$A = \{1, 2, 3, 4\}$

$B = \{(1,1), (1,2), (2,1), (2,2), (2,3), (2,4)\}$

Equivalence classes →

- Equivalence classes in the set of all elements of equivalence relation.

$[0]$ $\{x \in Z, (x-0)$ is divisible by $4\}$

$[0] = \{\ldots\ldots -12, -8, -4, 0, 4, 8, 12 \ldots\ldots\}$

$[1] \rightarrow \{x \in Z, (x-1)$ is divisible by $4\}$

$[1] = \{\ldots\ldots -11, -7, -3, 1, 5, 9, 13 \ldots\ldots\}$

Sum of n terms → $\frac{n(n+1)}{2}$

Sum of n^2 terms → $\frac{n(n+1)(2n+1)}{6}$

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