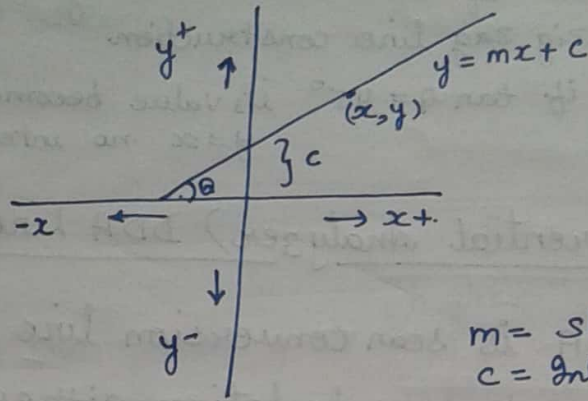


27-07-18



$m = \text{slope} = \tan \theta$   
 $c = \text{Interception Y axis}$

Line Equation is denoted by ' $y = mx + c$ ' where ' $m$ ' is the slope of line and ' $c$ ' is the intercept on the ' $y$ ' axis, such line passing through the point  $x, y$ . If line makes  $45^\circ$  angle on the  $x$  axis. It will pass through the origin. The equation of that line will be  $y = x$ .

Line Drawing Algorithm

1. Direct Method:

①.  $m = 1, c = 1$   
 $y = mx + c$

$x = 1$	$y = 1 \times 1 + 1 = 2$	$(1, 2)$
$x = 2$	$y = 1 \times 2 + 1 = 3$	$(2, 3)$
$x = 3$	$y = 1 \times 3 + 1 = 4$	$(3, 4)$

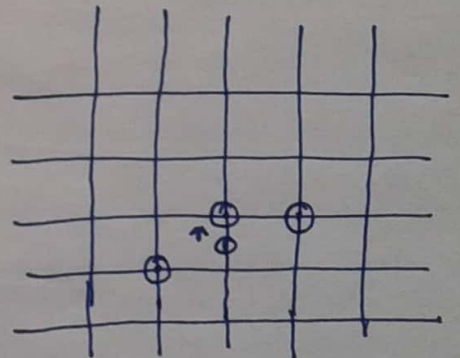


fig: ②

②. if  $m = .5, c = \frac{1}{2}$

$x = 1$	$y = .5 \times 1 + \frac{1}{2} = 1$	$(1, 1)$
$x = 2$	$y = .5 \times 2 + \frac{1}{2} = 1.5$	$(2, 1.5) = (2, 2)$
$x = 3$	$y = .5 \times 3 + \frac{1}{2} = 2$	$(3, 2)$
$x = 4$	$y = .5 \times 4 + \frac{1}{2} = 2.5$	$(4, 2.5) = (4, 3)$

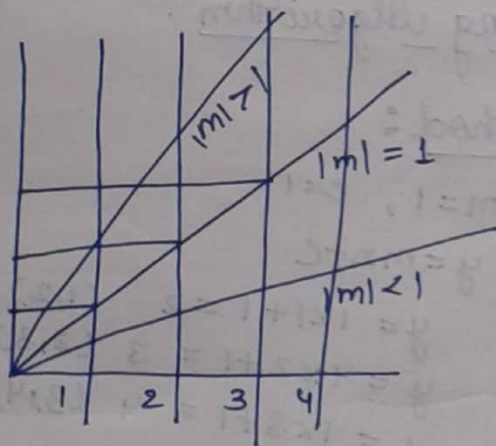
- Demerits -
- ① Floating numbers deviates location of pixel.
  - ② Lot of calculations (Heavy calculation)
  - ③ zig zag line construction

if  $\tan \theta = 45^\circ$  its value becomes 1 then  $y = x$  no interception.

28-07-2018

## 2. (Digital Differential Analyzer) DDA Line Drawing Algorithm

DDA is scan conversion line drawing algorithm, based on calculation either  $\Delta x$  or  $\Delta y$ . It generates a line using simultaneously incremental of  $\Delta x$  or  $\Delta y$  and calculate corresponding values of  $x$  and  $y$ . Such approach is characterized by performing calculation at each step using result from the preceding step. There may be following situations regarding the slope of line.



1.  $|m| = 1$  :

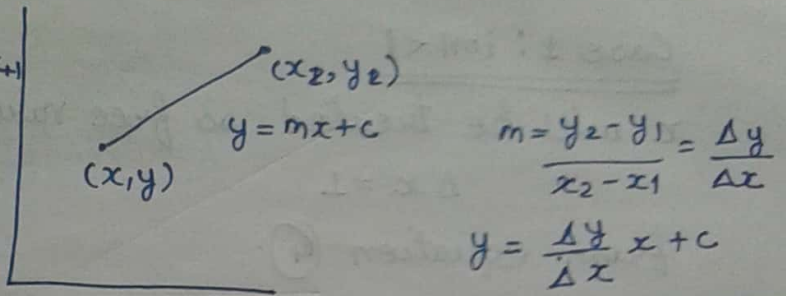
When the line pass through the origin and change in  $y$  will be equals to change in  $x$ .

2.  $|m| < 1$  :

If this case, the change in  $x$  is higher than change in  $y$ ,  $x$  would be treated as free variable and  $\Delta x = 1$

Geometrical Aspects :

(Next Position)  $K, K+1$   
 $x_1 \rightarrow$  Position of pixel  
 $x_2 \rightarrow$  Increment of pixel position



ii)  $|m| > 1$  :

In this case the change in  $y$  is <sup>higher</sup> than the change in  $x$ ,  $y$  would be treated as free variable and  $\Delta y = \pm$

If a line pass through a point  $(x, y)$ , the equation of that line will be

$$y = mx + c \quad \text{--- (1)}$$

if line pass through point  $(x_K, y_K)$

$$y_K = mx_K + c \quad \text{--- (2)}$$

Next position will  $(x_{K+1}, y_{K+1})$

$$y_{K+1} = mx_{K+1} + c \quad \text{--- (3)}$$

(3) - (2) Subtracting

$$y_{K+1} - y_K = (mx_{K+1} + c) - (mx_K + c)$$

$$y_{K+1} - y_K = mx_{K+1} + \cancel{c} - mx_K - \cancel{c}$$

$$y_{K+1} - y_K = m(x_{K+1} - x_K)$$

$$\boxed{\Delta y = m \Delta x} \quad \text{--- (4)}$$

Case 1:  $|m| < 1$

$x$  will be treated as free variable

$$\Delta x = 1$$

from equation (4)

$$\Delta y = m$$

$$\boxed{y_{k+1} - y_k = m} \rightarrow y_{k+1} = y_k + m$$

Case 2:  $|m| > 1$

$y$  will be treated as free variable

$$\Delta y = 1$$

from equation (4)

$$1 = m \Delta x$$

$$\Delta x = \frac{1}{m}$$

$$x_{k+1} - x_k = \frac{1}{m}$$

$$\boxed{x_{k+1} = x_k + \frac{1}{m}}$$

01.08.2018

Tuesday

## DDA Line Drawing Algorithm

Algorithm :-

Step 1: Read line and points  
 $(x_1, y_1)$  and  $(x_2, y_2)$

Step 2: Calculate  $dx = x_2 - x_1$ ,  $dy = y_2 - y_1$

Step 3: if  $abs(dy) > abs(dx)$   $\therefore abs \rightarrow absolute$   
then  $step = abs(dy)$

otherwise

$$step = abs(dx)$$

Step 4:  $x_{inc} = dx/step$ ,  $y_{inc} = dy/step$   $\therefore inc \rightarrow increment$

Step 5: set pixel ( $round(x)$ ,  $round(y)$ , color)

Step 6:  $x = x + x_{inc}$ ,  $y = y + y_{inc}$

set pixel ( $round(x)$ ,  $round(y)$ , color)

Repeat step 6 until  $x = x_2$ ,  $y = y_2$

Numerical: Draw a line by using DDA Line Drawing Algorithm  
its ends point are  $(1, 10)$  and  $(5, 2)$

$$x_1 = 1 \quad x_2 = 5$$

$$y_1 = 10 \quad y_2 = 2$$

$$dx = 5 - 1 = 4, \quad abs(dx) = 4$$

$$dy = 2 - 10 = -8, \quad abs(dy) = 8 \quad (\text{modulation})$$

$$\text{Hence } abs(dy) < abs(dx) \quad \therefore step = 8$$

$$x_{inc} = \frac{dx}{step} = \frac{4}{8} = 0.5$$

$$y_{inc} = \frac{dy}{step} = \frac{-8}{8} = -1$$

$(x, y)$	<u>round</u>
$(1, 10)$	$(1, 10)$
$(1.5, 9)$	$(2, 9)$
$(2, 8)$	$(2, 8)$
$(2.5, 7)$	$(3, 7)$
$(3, 6)$	$(3, 6)$
$(3.5, 5)$	$(4, 5)$
$(4, 4)$	$(4, 4)$
$(4.5, 3)$	$(5, 3)$
$(5, 2)$	$(5, 2)$

Algori  
point  
satisfie

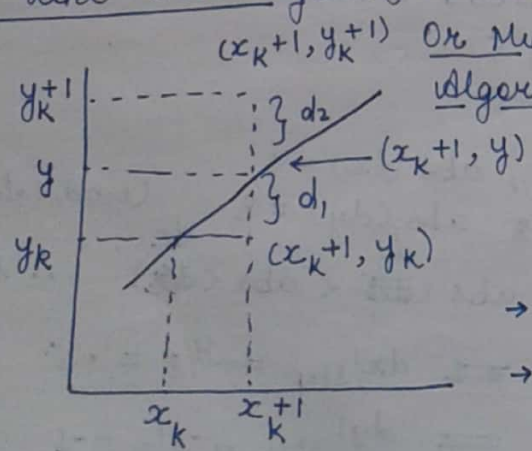
Advantage :

1. DDA Algorithm is a faster method for calculating pixel position than the direct method.
2. It eliminates the calculations upto some extent.

Disadvantages :

1. Rounding operation and floating point numbers still exist that actual line drawing is not possible.
2. The stair effect on the line can be seen in this algorithm.

Bresenham's Line Drawing Algorithm.



Or Midpoint Line Drawing Algorithm

- $\rightarrow d_1 > d_2 \rightarrow (x_{k+1}, y_{k+1})$
- $\rightarrow d_1 < d_2 \rightarrow (x_{k+1}, y_k)$

$p_k =$

Algorithm:

02-08-2018  
Thursday

point  $(x_{k+1}, y)$  is on the line so it must be satisfied line equation  $y = mx + b$ .

$$y = m(x_{k+1}) + b \quad \text{--- ①}$$

$$d_1 = y - y_k$$

$$d_1 = [m(x_{k+1}) + b] - y_k \quad \text{--- ②}$$

$$d_2 = y_{k+1} - y$$

$$d_2 = [y_{k+1}] - [m(x_{k+1}) + b] \quad \text{--- ③}$$

03-08-2018  
Friday

$$d_1 = m(x_{k+1}) + b - y_k$$

$$d_2 = (y_{k+1}) - m(x_{k+1}) - b$$

$$\text{Hence } d_1 - d_2 = [m(x_{k+1}) + b - y_k] - [(y_{k+1}) - m(x_{k+1}) - b]$$

$$= \underline{m(x_{k+1}) + b - y_k} - (y_{k+1}) + \underline{m(x_{k+1}) + b}$$

$$= 2m(x_{k+1}) + b - y_k - y_{k+1} + b$$

$$d_1 - d_2 = 2m(x_{k+1}) - 2y_k + 2b - 1$$

$$= 2 \times \frac{\Delta y}{\Delta x} \times (x_{k+1}) - 2y_k + 2b - 1$$

$$= \frac{2 \times \Delta y (x_{k+1}) - 2 \Delta x y_k + 2 \Delta x b - \Delta x}{\Delta x}$$

$$p_k = \Delta x (d_1 - d_2) = 2 \Delta y x_k + 2 \Delta y - 2 \Delta x y_k + 2b \Delta x - \Delta x$$

$p_k$  = decision parameter at  $k^{\text{th}}$  position

$$= 2 \Delta y x_k - 2 \Delta x y_k + 2 \Delta y + 2b \Delta x - \Delta x$$

$$= 2 \Delta y x_k - 2 \Delta x y_k + 2 \Delta y + \Delta x (2b - 1)$$

$$2\Delta y + \Delta x(2b-1) = K$$

$K$  is the constant which is independent to the pixel position.

$$p_k = 2\Delta y x_k - 2\Delta x y_k + K \quad \text{--- (4)}$$

where  $K = 2\Delta y + \Delta x(2b-1)$

at  $k+1$ th position equation will be

$$p_{k+1} = 2\Delta y x_{k+1} - 2\Delta x y_{k+1} + K \quad \text{--- (5)}$$

Subtracting (5) - (4) {changing equation 4 sign}

$$p_{k+1} - p_k = 2\Delta y x_{k+1} - 2\Delta x y_{k+1} + K - (2\Delta y x_k - 2\Delta x y_k + K)$$

$$= 2\Delta y (x_{k+1} - x_k) - 2\Delta x (y_{k+1} - y_k) \quad \text{--- (6)}$$

Case 1:  $|m| < 1$

$$\Delta x = 1$$

$$x_{k+1} - x_k = 1$$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x (y_{k+1} - y_k)$$

~~where  $p_k < 0$  or  $d_1 < d_2 < 0$~~

if  $p_k < 0$

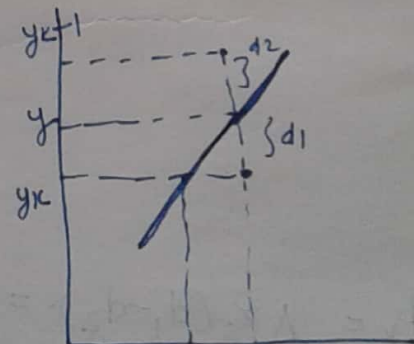
position  $(x_{k+1}, y_k)$

$$y_{k+1} - y_k = 0$$

$$p_{k+1} = p_k + 2\Delta y$$

else

~~$p_k$~~   
 $d_1$  greater  $p_k +$   
 $d_2$  greater  $p_k -$



$$p_k = d_1 - d_2$$

$p_k < 0$  means  $d_2 > d_1$

$p_k > 0$  means  $d_1 > d_2$



position will be ( ~~$x_k$~~   $x_{k+1}, y_{k+1}$ )

$$y_{k+1} - y_k = 1$$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

Case 2:  $m > 1$

08-08-2018  
Wednesday

$$y_{k+1} - y_k = 1$$

$$p_{k+1} = p_k + 2\Delta y (x_{k+1} - x_k) - 2\Delta x (y_{k+1} - y_k)$$

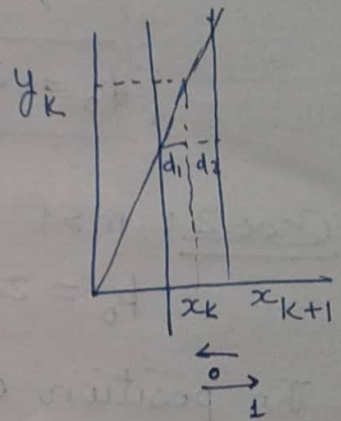
$$p_{k+1} = p_k + 2\Delta y (x_{k+1} - x_k) - 2\Delta x$$

If  $p_k < 0$

the point will be  $(x_k, y_{k+1})$

$$x_{k+1} - x_k = 0$$

$$p_{k+1} = p_k - 2\Delta x$$



else

the point will be  $(x_{k+1}, y_{k+1})$

$$x_{k+1} - x_k = 1$$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

First Decision Parameter at position  $(x_0, y_0)$

Case  $m < 1$  :-

From equation 4

$$p_k = 2\Delta y x_k - 2\Delta x y_k + k$$

$$p_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + \Delta x (2b - 1)$$

$$y = mx + b$$

So.  $b = (y_0 - mx)$

$$p_0 = 2\Delta y x_0 - 2x_0 y_0 + 2\Delta y + \Delta x (2(y_0 - mx_0) - 1)$$

$$m = \frac{\Delta y}{\Delta x}$$

$$p_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + \Delta x (2(y_0 - \frac{\Delta y}{\Delta x} x_0) - 1)$$

$$p_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + \Delta x \left( \frac{2\Delta x y_0 - \Delta y x_0 - \Delta x}{\Delta x} \right)$$

$$p_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + 2\Delta y - 2\Delta y x_0 - \Delta x$$

$$\boxed{p_0 = 2\Delta y - \Delta x}$$

Case 2:  $m > 1$

$$p_0 = 2\Delta x - \Delta y$$

The position of  $x$  and  $y$  will be changed and equation will be derived.

Algorithm:

Step 1: Input line end points  $(x_1, y_1)$  and  $(x_2, y_2)$ .

Step 2: Calculate

$$\Delta y = y_2 - y_1$$

$$\Delta x = x_2 - x_1$$

Case 1:  $m < 1$

$$p_0 = 2\Delta y - \Delta x$$

Step 3: if  $(p_k < 0)$

$$p_{k+1} = p_k + 2\Delta y$$

otherwise

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

Step 4: Repeat step 3  $\Delta x$  times

Step 5: Case 2:  $m > 1$

$$p_0 = 2\Delta x - \Delta y$$

if  $(p_k < 0)$

$$p_{k+1} = p_k - 2\Delta x$$

Otherwise

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

Step 6: Repeat step 5  $\Delta y$  times.

09-08-2018

Thursday

Draw a line by using Bresenham's algorithm its two end point on  $(10, 5)$  and  $(15, 9)$ .

$$x_1 = 10$$

$$x_2 = 15$$

$$y_1 = 5$$

$$y_2 = 9$$

$$\Delta x = x_2 - x_1 = 15 - 10 = 5$$

$$\Delta y = y_2 - y_1 = 9 - 5 = 4$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{4}{5} = .8$$

Here,  $m < 1$

$$\begin{aligned} p_0 &= 2\Delta y - \Delta x \\ &= 2 \times 4 - 5 \\ &= 3 \end{aligned}$$

Now, for getting next point, we calculate next decision parameter

$$\begin{aligned} p_1 &= p_0 + 2\Delta y - 2\Delta x \\ &= 3 + 2 \times 4 - 2 \times 5 \\ &= 3 + 8 - 10 \\ &= 1 \end{aligned}$$

Now,  $P_k$  is greater than 0, so applying the equation

$$P < 0$$

$$(x_{k+1}, y_k)$$

else  $P > 0$

$(x_{k+1}, y_{k+1}) \rightarrow$  This equation will be used, because our else parts will become true, so coordinate become

$$(11, 6)$$

$$\begin{aligned} P_2 &= P_1 + 2\Delta y - 2\Delta x \\ &= 1 + 2 \times 4 - 2 \times 5 \\ &= 1 + 8 - 10 \\ &= -1 \end{aligned}$$

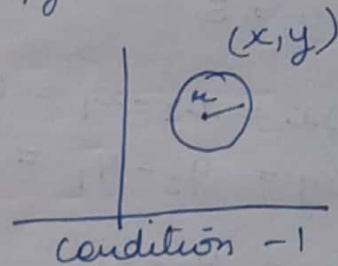
## Circle Drawing Algorithm or

### Mid point Circle Drawing Algorithm

When a circle passing through a point  $(x, y)$  what will be the equation:

$$x^2 + y^2 = r^2$$

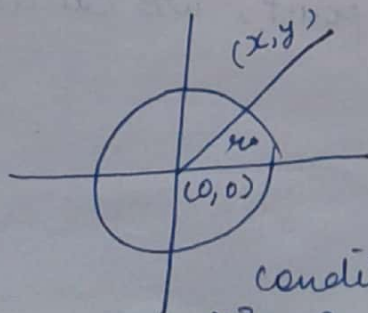
← not satisfied



### Explanation

When the circle passing through point  $(x, y)$  and center of the circle on the origin, radius of circle is  $r$ , the equation will be  $x^2 + y^2 = r^2$ .

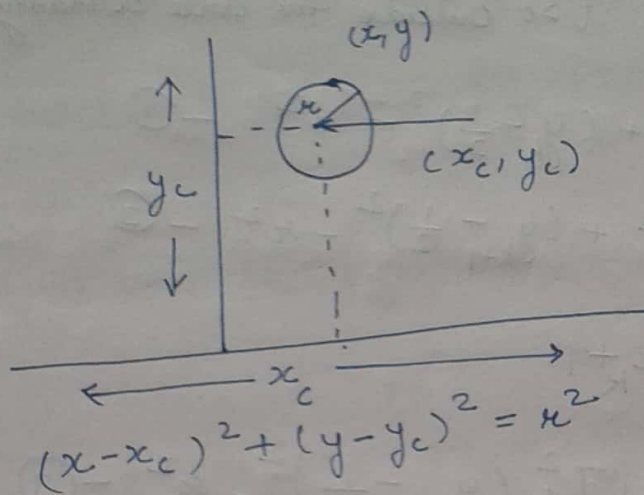
↑ satisfies.



$$\begin{aligned} &\text{condition:2} \\ &x^2 + y^2 = r^2 \end{aligned}$$

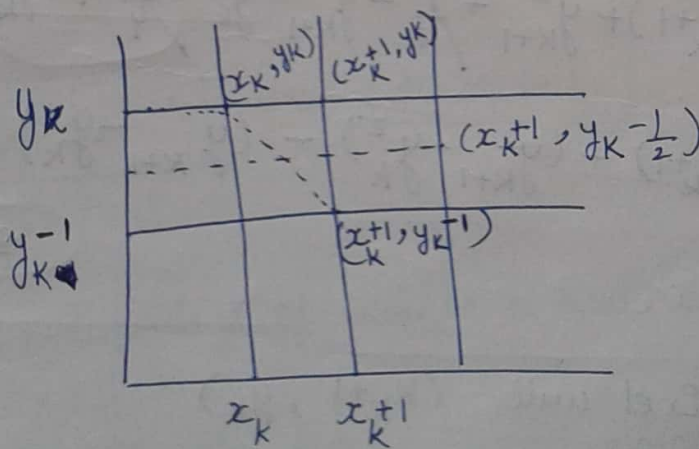
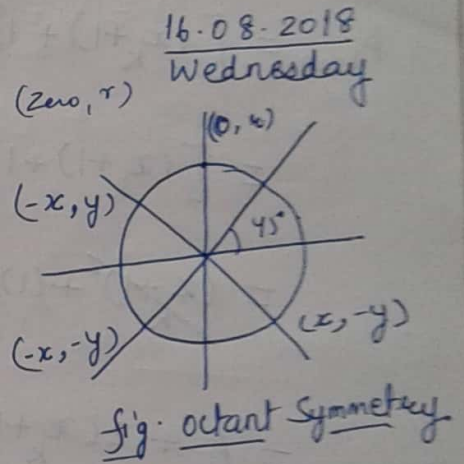
## Explanation

When the circle moves from the origin and centre co-ordinates are  $x_c$  and  $y_c$  in that case, circle equation will be  $(x-x_c)^2 + (y-y_c)^2 = r^2$



## Octant Symmetry of Circle

When we divide a circle in eight equal parts. This kind of symmetry is called Octant Symmetry of Circle. In Circle Drawing Algorithm, we generate the algorithm for single part and apply for remaining seven parts.



$$f_{\text{circle}}(x, y) = x^2 + y^2 - r^2$$

$$f_{\text{circle}}(x, y) = \begin{cases} < 0 & \text{point } (x, y) \text{ inside the circle boundary} \\ = 0 & \text{on circle boundary} \\ > 0 & \text{outside the circle boundary} \end{cases}$$

$$p_k = (x_k + 1)^2 + (y_k - \frac{1}{2})^2 - r^2 \quad \text{--- (1)}$$

$$p_{k+1} = (x_{k+1} + 1)^2 + (y_{k+1} - \frac{1}{2})^2 - r^2 \quad \text{--- (2)}$$

$$x_{k+1} = x_k + 1$$

$$p_{k+1} = [(x_k + 1) + 1]^2 + (y_{k+1} - \frac{1}{2})^2 - r^2 \quad \text{--- (3)}$$

$$= [(x_k + 1) + 1]^2 + (y_{k+1} - \frac{1}{2})^2 - [(x_k + 1)^2 + (y_k - \frac{1}{2})^2 - p_k]$$

$$= \cancel{(x_k + 1)^2} + (1)^2 + 2(x_k + 1) + (y_{k+1} - \frac{1}{2})^2 + p_k - \cancel{(x_k + 1)^2} - (y_k - \frac{1}{2})^2$$

$$p_{k+1} = p_k + 2(x_k + 1) + y_{k+1}^2 + \frac{1}{4} - 2 \times y_{k+1} \times \frac{1}{4} - [y_k^2 + \frac{1}{4} - \cancel{2 \times y_k \times \frac{1}{4}}] + 1$$

$$p_{k+1} = p_k + 2(x_k + 1) + y_{k+1}^2 + \frac{1}{4} - y_{k+1} - \frac{y_k^2}{4} - \frac{1}{4} + y_k + 1$$

$$p_{k+1} = p_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

29-08-2018

if  $p_k < 0$

the next pixel will  $(x_{k+1}, y_k)$

$$y_{k+1} = y_k$$

$$p_{k+1} = p_k + 2(x_k + 1) + (y_k^2 - y_k^2) - (y_k - y_k) + 1$$

$$p_{k+1} = p_k + 2(x_k + 1) + 1$$

$$\text{or } p_{k+1} = p_k + 2x_{k+1} + 1$$

else

next point will be  $(x_k + 1, y_k - 1)$

$$y_{k+1} = y_k - 1$$

$$p_{k+1} = p_k + 2(x_k + 1) + ((y_k - 1)^2 - y_k^2) - (y_k - 1 - y_k) + 1$$

$$p_{k+1} = p_k + 2(x_k + 1) + (y_k^2 + 1 - 2y_k - y_k^2) + 1 + 1$$

$$= p_k + 2x_k + 2 + 1 - 2y_k + 2$$

$$p_{k+1} = p_k + 2(x_k + 1) - 2(y_k + 1) + 1$$

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

First Decision Parameter at point  $(0, r)$

$$(1, r - \frac{1}{2})$$

$$p_0 = (1)^2 + (r - \frac{1}{2})^2 - r^2$$

$$= 1 + r^2 + \frac{1}{4} - 2r \times \frac{1}{2} - r^2$$

$$p_0 = \frac{5}{4} - r \quad \text{or} \quad p_0 = 1 - r$$

Algorithm:

Step 1: Input radius of circle  $r$  and center point coordinates  $(x_c, y_c)$

Step 2: Initial decision parameter at point  $(0, r)$

$$p_0 = \frac{5}{4} - r$$

Step 3 : if  $(p_k < 0)$

next position will be  $(x_{k+1}, y_k)$

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

else

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

Step 4 : Determine Symmetry points in the seven other octants.

Step 5 :  $x = x + x_c$

$y = y + y_c$

Step 6 : Repeat step 3 through 5 until  $x \geq y$

30.08.2018

Thursday.

### Ellipse Drawing Algorithm

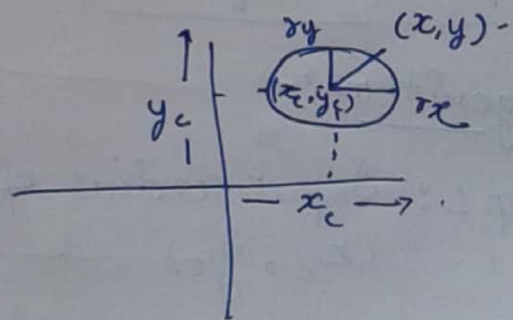
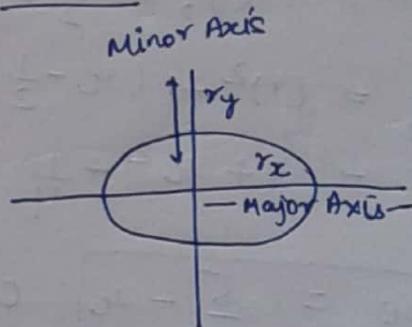
### or Midpoint Ellipse Drawing Algorithm

$$\frac{(x-x_c)^2}{k_x^2} + \frac{(y-y_c)^2}{k_y^2} = 1$$

$$\frac{x^2}{k_x^2} + \frac{y^2}{k_y^2} = 1$$

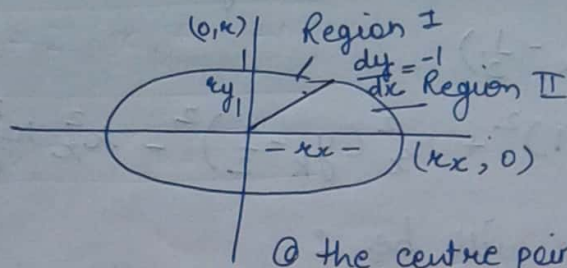
or.  $x^2 k_y^2 + y^2 k_x^2 = k_x^2 k_y^2$

after cross multiplication





## Quadrant Symmetry of Ellipse



@ the centre point the value of  $\frac{dy}{dx} = -1$ .

The condition on the center point of ellipse:

$$\frac{d}{dx} (k_y^2 x^2) + \frac{d}{dx} (k_x^2 y^2) = 0.$$

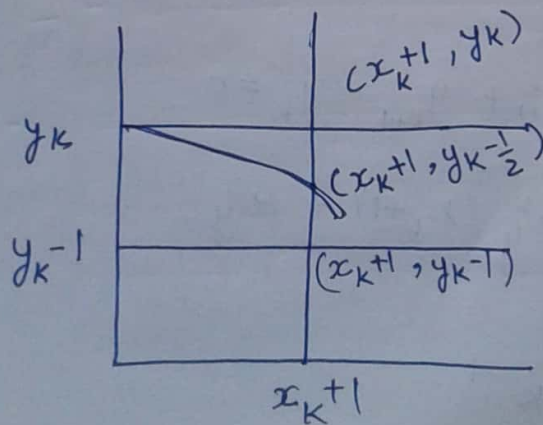
$$2xk_y^2 + 2k_x^2 y \frac{dy}{dx} = 0.$$

$$2k_x^2 y \frac{dy}{dx} = -2xk_y^2$$

$$\frac{dy}{dx} = \frac{-2xk_y^2}{2yk_x^2} = -\frac{2xk_y^2}{2yk_x^2}$$

$$+1 = +\frac{2xk_y^2}{2yk_x^2}$$

$$\boxed{xk_y^2 = yk_x^2}$$



ellipse equation will satisfy

$$(x_{k+1}, y_k - \frac{1}{2})$$

$$p_{1k} = \kappa_y^2 (x_{k+1})^2 + \kappa_x^2 (y_k - \frac{1}{2})^2 - \kappa_x^2 \kappa_y^2$$

The next point will be :

$$(x_{k+1} + 1, y_{k+1} - \frac{1}{2})$$

$$\begin{aligned} p_{1k+1} &= \kappa_y^2 ((x_{k+1}) + 1)^2 + \kappa_x^2 (y_{k+1} - \frac{1}{2})^2 - \kappa_x^2 \kappa_y^2 \\ &= \kappa_y^2 [(x_{k+1})^2 + 1 + 2(x_{k+1})] + \kappa_x^2 (y_{k+1} - \frac{1}{2})^2 - \kappa_x^2 \kappa_y^2 \end{aligned}$$

$$p_{1k+1} = \kappa_y^2 [(x_{k+1})^2 + 1 + 2(x_{k+1})] + \kappa_x^2 (y_{k+1} - \frac{1}{2})^2 - [\kappa_y^2 (x_k + 1)^2 + \kappa_x^2 (y_k - \frac{1}{2})^2 - p_{1k}]$$

$$p_{1k+1} = \kappa_y^2 (x_k + 1)^2 + \kappa_y^2 + 2\kappa_y^2 (x_k + 1) + \kappa_x^2 (y_{k+1} - \frac{1}{2})^2 - \kappa_y^2 (x_k + 1)^2 - \kappa_x^2 (y_k - \frac{1}{2})^2 + p_{1k}$$

$$p_{1k+1} = p_k + 2\kappa_y^2 (x_k + 1) + \kappa_y^2 + \kappa_x^2 ((y_{k+1} - \frac{1}{2})^2 - (y_k - \frac{1}{2})^2)$$

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if  $(p_{1k} < 0)$  at this point  $y_{k+1} - y_k = 0$

$$p_{1k+1} = p_{1k} + 2\kappa_y^2 (x_k + 1) + \kappa_y^2$$

otherwise

$$y_{k+1} - y_k = 1$$

$$P_{1k+1} = P_{1k} + 2x^2 y x_{k+1} - 2x^2 x y_{k+1} + x^2 y$$

First decision parameter in Region - I at point  $(0, xy)$

$$\left(\frac{1}{x}, \frac{xy - \frac{1}{2}}{y}\right)$$

$$P_{10} = x^2 y + x^2 x \left(xy - \frac{1}{2}\right)^2 - x^2 x x^2 y$$

$$P_{10} = x^2 y - x^2 x x^2 y + \frac{1}{4} x^2 x$$

In Region II

$$\text{Point will be } \left(\frac{x_k + \frac{1}{2}}{x}, \frac{y_k - 1}{y}\right)$$

$$P_{2k} = x^2 y \left(x_k + \frac{1}{2}\right)^2 + x^2 x \left(y_k - 1\right)^2 - x^2 x x^2 y$$

$$\text{Next point will be } \left(\frac{x_{k+1} + \frac{1}{2}}{x}, \frac{y_{k+1} - 1}{y}\right)$$

$$P_{2k+1} = x^2 y \left(x_{k+1} + \frac{1}{2}\right)^2 + x^2 x \left(y_{k+1} - 1\right)^2 + x^2 x x^2 y$$

$$P_{2k+1} = P_{2k} - 2x^2 x \left(y_k - 1\right) + x^2 x + x^2 y \left[\left(x_k + \frac{1}{2}\right)^2 - \left(x_{k+1} + \frac{1}{2}\right)^2\right]$$

If  $(P_{2k} > 0)$

$$P_{2k+1} = P_{2k} - 2x^2 x y_{k+1} + x^2 x$$

Otherwise

$$P_{2k+1} = P_{2k} - 2x^2 x y_{k+1} + x^2 x + 2x^2 y x_{k+1} + x^2 x$$

First decision parameter in Region - II

$$P_{20} = x^2 y \left(x_0 + \frac{1}{2}\right)^2 + x^2 x \left(y_0 - 1\right)^2 - x^2 x x^2 y$$

## Algorithm

Step 1: Input centre-point of ellipse that is  $(x_c, y_c)$  and radius of ellipse is  $r$ .

→ First point will be  $(0, r_y)$

Step 2: Calculate first decision parameter in Region-I

$$P_{10} = r^2 y - r^2 x r^2 y + \frac{1}{4} r^2 x$$

Step 3: Calculate

if  $(P_{1k} < 0)$

$$P_{1k+1} = P_{1k} + 2r^2 y (x_{k+1}) + r^2 y$$

or

$$P_{1k+1} = P_{1k} + 2r^2 y x_{k+1} + r^2 y$$

otherwise

$$P_{1k+1} = P_{1k} + 2r^2 y x_{k+1} - 2r^2 x y_{k+1} + r^2 y$$

until  $2r^2 y x \geq 2r^2 x y$

Step 4: First decision parameter for Region II

$$P_{20} = r^2 y (x_0 + \frac{1}{2})^2 + r^2 x (y_0 - 1)^2 - r^2 x r^2 y$$

Step 5: if  $(P_{2k} > 0)$

$$P_{2k+1} = P_{2k} - 2r^2 x y_{k+1} + r^2 x$$

otherwise

$$P_{2k+1} = P_{2k} - 2r^2 x y_{k+1} + 2r^2 y x_{k+1} + r^2 x$$

Step 6: Determine Symmetry for other three quadrant.

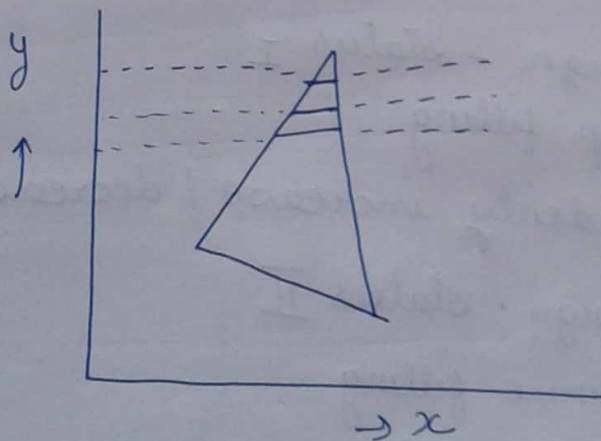
Step 7:  $x = x + x_c$ ,  $y = y + y_c$  until  $2x^2yx \geq 2x^2xy$ .

## Filling Algorithm

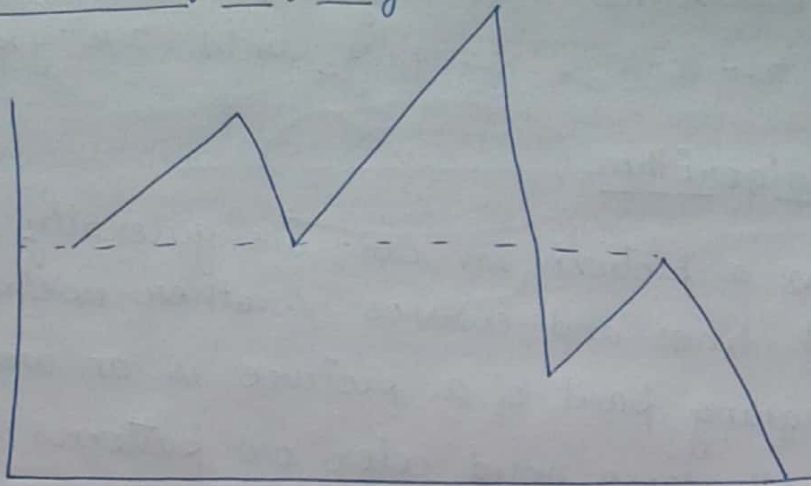
To draw a picture in CG, we generally use points, straight lines and curves. Another useful construct for designing part of a picture is an area that is filled with some solid color or patterns. The filling algorithm finds interior part of the polygon and assigned a color or action. There are some methods which used to fill in the algorithm.

### 1. Scan line polygon filling Algorithm

This algorithm used for solid filling of a polygon when scan-line intersect the polygon  $H$  by making moving left to right it start the filling when again intersect with another  $H$  it stop the filling.



## Another condition for filling

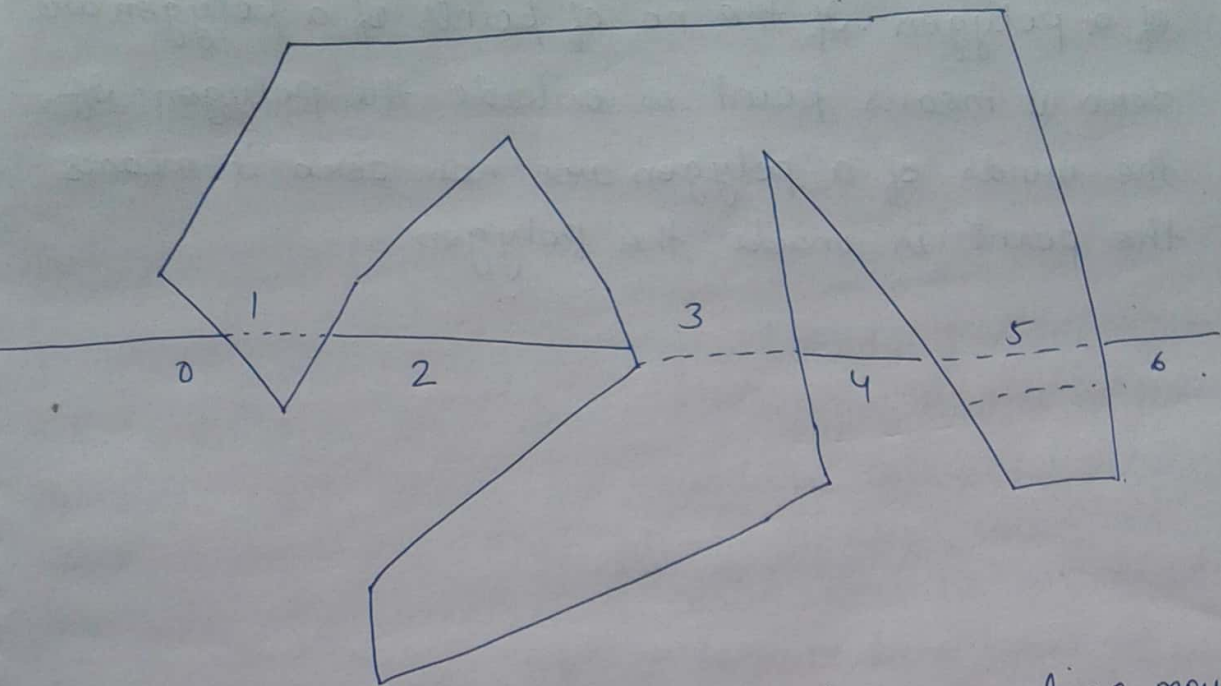


If the scan line passing through the vertex of intersect of two ellipse edges, we follow the following algorithm:

1. When the scan line intersects with edge of polygon start the filling and is again intersect with edge of polygon it stop the filling.
2. If scan line passes through the intersect of two edges then we require following additional processing
  - i) Value of  $y$  monotonically increase / decrease  
assign - status I  
stop filling
  - ii) Value of suddenly increase / decrease  
Assign - status II  
continue filling

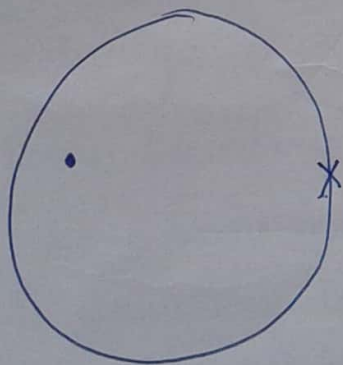
② inside  $\neq$  outside test  
filling algorithm needs to identify intersect object region of an object.

### i) odd - even Rule

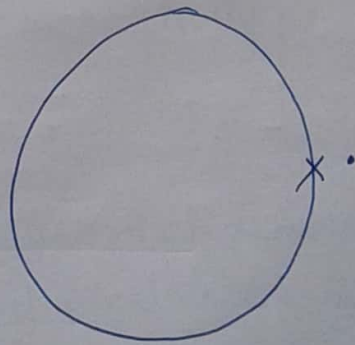


It is also called parity rule, the scan line move from left to right if the no. of intersection is odd it means scan line inside the polygon and if it even then scan line outside the polygon.

### ii) Non-Zero Winding Rule

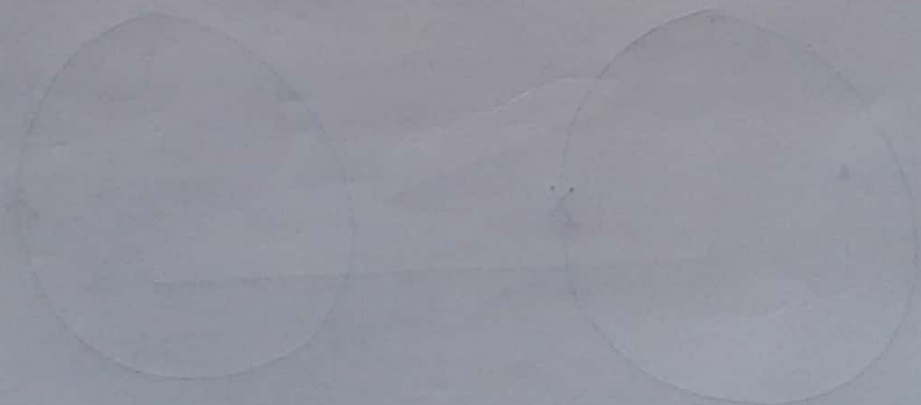


winding 1



winding 0

In this algorithm we count the number of winds of a polygon. If the no. of winds of a polygon are zero it means point is outside the polygon. If the winds of a polygon are non-zero it means the point is inside the polygon.





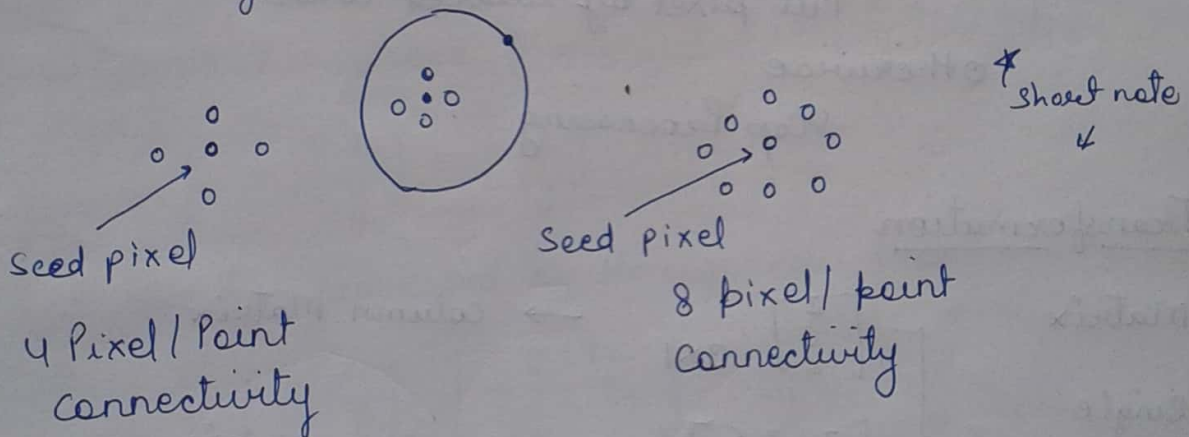
11. Oct. 2018

## Area Filling Algorithm

1. Boundary Fill Algorithm
2. Flood Fill Algorithm

### Boundary Fill Algorithm

In this algorithm, we store the boundary color and take a point or pixel inside the polygon that is called the seed point, check the boundary color and pixel color is different, then fill the desired color in the pixel. This process will continue until we find the boundary color. We can follow two types of pattern for filling the polygon, one is the four point connectivity and second is the eight point connectivity.



Step 1: Store boundary color

Step 2: Take a point inside polygon that is seed point

Step 3: If Boundary color and pixel color is different  
Fill pixel by desired color  
otherwise  
stop processing

## 2. Flood Fill Algorithm

In this algorithm we store the background of the polygon, Take a point inside the polygon i.e called seed point, if the background color and the pixel color is the same it means the pixel is unfilled and filled the desired color. Use the four point connectivity or eight point connectivity to continue the process

### Algorithm :

Step 1: Store background color

Step 2: Take a point inside the polygon that is seed point

Step 3: If background color and pixel color is same.  
Fill pixel by desired color  
otherwise  
Step Processing

## Transformation

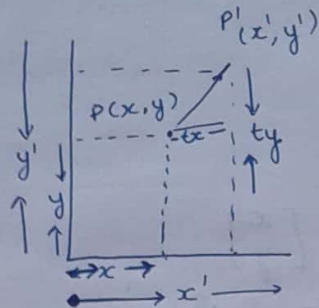
Matrix  $\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} 3 \times 1 \rightarrow$  column Matrix

Single Dimension  $\begin{bmatrix} 2 & 3 & 5 \end{bmatrix} 1 \times 3 \rightarrow$  row Matrix.

Two Dimension  $\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & 5 \\ 8 & 9 & 2 \end{bmatrix} 3 \times 3$

## 1. Translation

Moving an object / pixel from one position / location to another that is called Translation.



P is the point having the coordinate  $x, y$ , it is translated to new position  $P'$  having the coordinate  $x', y'$ .  $tx$  is the translation factor on the  $x$  axis and  $ty$  is the translation factor on  $y$  axis.

$$x' = x + tx$$

$$y' = y + ty$$

$$P' = P + T \quad \text{where } P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad P = \begin{bmatrix} x \\ y \end{bmatrix} \quad T = \begin{bmatrix} tx \\ ty \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$$

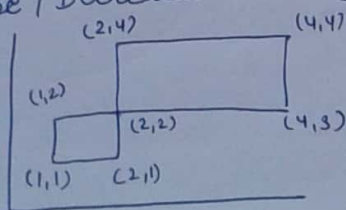
It can be expressed by Homogeneous coordinate System.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x+tx \\ y+ty \\ 1 \end{bmatrix} \text{ Product}$$

## 2. Scaling

Increase / Decrease the size of an object is called scaling.



Initial points are  $(x, y)$  New points are  $(x', y')$

$S_x$  = Scaling factor on x Axis

$S_y$  = Scaling factor on y Axis

$$x' = x \cdot S_x$$

$$y' = y \cdot S_y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous Coordinate System

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$